α-Alum [KAl(SO$_4$)$_2$ · 12H$_2$O, $H4_{13}$] Structure: AB24CD28E2_cP224_205_a_4d_b_2c4d_c-001

This structure originally had the label `AB24CD28E2_cP224_205_a_4d_b_2c4d_c`. Calls to that address will be redirected here.

https://aflow.org/p/MHB8

https://aflow.org/p/AB24CD28E2_cP224_205_a_4d_b_2c4d_c-001
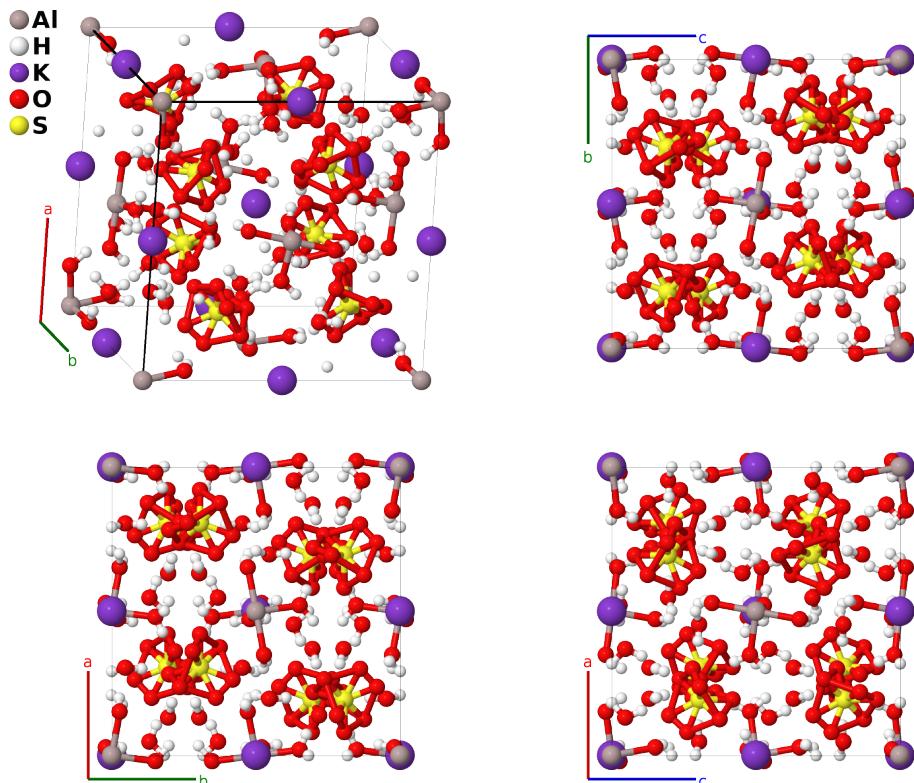
Prototype	AlH$_{24}$KO$_{20}$S$_2$
AFLOW prototype label	AB24CD28E2_cP224_205_a_4d_b_2c4d_c-001
***Strukturbericht* designation**	$H4_{13}$
Mineral name	α-alum
ICSD	280547
Pearson symbol	cP224
Space group number	205
Space group symbol	$Pa\bar{3}$
AFLOW prototype command	aflow --proto=AB24CD28E2_cP224_205_a_4d_b_2c4d_c-001 --params=$a, x_3, x_4, x_5, x_6, y_6, z_6, x_7, y_7, z_7, x_8, y_8, z_8, x_9, y_9, z_9, x_{10}, y_{10}, z_{10}, x_{11}, y_{11}, z_{11}, x_{12}, y_{12}, z_{12}, x_{13}, y_{13}, z_{13}$

Other compounds with this structure

$NH_4Al(SO_4)_2 \cdot 12H_2O$, $KCr(SO_4)_2 \cdot 12H_2O$, $RbAl(SO_4)_2 \cdot 12H_2O$, $KAl(SeO_4)_2 \cdot 12H_2O$

- The alums have the general formula $AB(XO_4)_2 \cdot 12H_2O$, where A is a monovalent ion, B is a trivalent ion, and X is a chalcogen. In most cases atom B is aluminum and atom X is sulfur, leading to the name alum.

- All alums have their room-temperature form in space group $Pa\overline{3}$ #205, but the bonding between the A and B ions and the XO_4 complex can be quite different.

- (Lipson, 1935ab) described three general forms of alum based on the sizes of the monovalent ions. Each of these forms was given a *Strukturbericht* designation by (Gottfried, 1937):

  - α-alum, with intermediate sized ions, prototype $KAl(SO_4)_2 \cdot 12H_2O$, $H4_{13}$ (this structure),
  - β-alum, with large ions, prototype $(NH_3CH_3)Al(SO_4)_2 \cdot 12H_2O$, $H4_{14}$, and
  - γ-alum, with small ions, prototype $NaAl(SO_4)_2 \cdot 12H_2O$, $H4_{15}$.

- This classification scheme is not compete, *e.g.*, (Ledsham, 1968) points out that $NaCr(SO_4)_2 \cdot 12H_2O$ does not fit into any of these categories, and that the actual structure depends on the combination of monovalent and trivalent ions.

- As noted above, the $Pa\overline{3}$ structures of alum are the room temperature form. As the temperature decreases the alum structure may transform. For example, in the temperature range 150-170K the β-alum $(NH_3CH_3)Al(SO_4)_2 \cdot 12H_2O$ transforms into an orthorhombic structure with fully ordered NH_3CH_3 ions.

- (Ewald, 1931) designated the α-alum structure determined by (Cook, 1927) as $H4_2$, but when the superior structural determination of (Beevers, 1934) appeared this type was abandoned, and the new structure was given the designation $H4_{13}$. (Nyburg, 2000) were able to determine the positions of the hydrogen atoms, so we use their improved structure as our prototype.

- The oxygen atoms around the sulfur are statistically distributed in two ways: the probability of the O-I and O-III sites being occupied is 78.7%, while the probability of the O-II/O-IV combination being occupied is 21.3%.

Simple Cubic primitive vectors

$$\mathbf{a_1} = a\,\hat{\mathbf{x}}$$
$$\mathbf{a_2} = a\,\hat{\mathbf{y}}$$
$$\mathbf{a_3} = a\,\hat{\mathbf{z}}$$
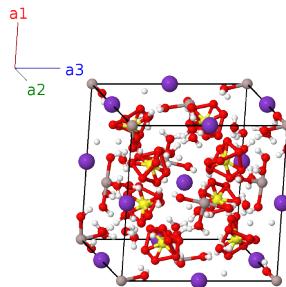
Basis vectors

		Lattice coordinates		Cartesian coordinates	Wyckoff position	Atom type
$\mathbf{B_1}$	=	0	=	0	(4a)	Al I
$\mathbf{B_2}$	=	$\frac{1}{2}\mathbf{a_1} + \frac{1}{2}\mathbf{a_3}$	=	$\frac{1}{2}a\,\hat{\mathbf{x}} + \frac{1}{2}a\,\hat{\mathbf{z}}$	(4a)	Al I
$\mathbf{B_3}$	=	$\frac{1}{2}\mathbf{a_2} + \frac{1}{2}\mathbf{a_3}$	=	$\frac{1}{2}a\,\hat{\mathbf{y}} + \frac{1}{2}a\,\hat{\mathbf{z}}$	(4a)	Al I
$\mathbf{B_4}$	=	$\frac{1}{2}\mathbf{a_1} + \frac{1}{2}\mathbf{a_2}$	=	$\frac{1}{2}a\,\hat{\mathbf{x}} + \frac{1}{2}a\,\hat{\mathbf{y}}$	(4a)	Al I
$\mathbf{B_5}$	=	$\frac{1}{2}\mathbf{a_1} + \frac{1}{2}\mathbf{a_2} + \frac{1}{2}\mathbf{a_3}$	=	$\frac{1}{2}a\,\hat{\mathbf{x}} + \frac{1}{2}a\,\hat{\mathbf{y}} + \frac{1}{2}a\,\hat{\mathbf{z}}$	(4b)	K I
$\mathbf{B_6}$	=	$\frac{1}{2}\mathbf{a_2}$	=	$\frac{1}{2}a\,\hat{\mathbf{y}}$	(4b)	K I

$$\mathbf{B_7} = \tfrac{1}{2}\,\mathbf{a}_1 = \tfrac{1}{2}a\,\hat{\mathbf{x}} \quad\text{(4b)} \quad\text{K I}$$

$$\mathbf{B_8} = \tfrac{1}{2}\,\mathbf{a}_3 = \tfrac{1}{2}a\,\hat{\mathbf{z}} \quad\text{(4b)} \quad\text{K I}$$

$$\mathbf{B_9} = x_3\,\mathbf{a}_1 + x_3\,\mathbf{a}_2 + x_3\,\mathbf{a}_3 = ax_3\,\hat{\mathbf{x}} + ax_3\,\hat{\mathbf{y}} + ax_3\,\hat{\mathbf{z}} \quad\text{(8c)} \quad\text{O I}$$

$$\mathbf{B_{10}} = -\left(x_3 - \tfrac{1}{2}\right)\mathbf{a}_1 - x_3\,\mathbf{a}_2 + \left(x_3 + \tfrac{1}{2}\right)\mathbf{a}_3 = -a\left(x_3 - \tfrac{1}{2}\right)\hat{\mathbf{x}} - ax_3\,\hat{\mathbf{y}} + a\left(x_3 + \tfrac{1}{2}\right)\hat{\mathbf{z}} \quad\text{(8c)} \quad\text{O I}$$

$$\mathbf{B_{11}} = -x_3\,\mathbf{a}_1 + \left(x_3 + \tfrac{1}{2}\right)\mathbf{a}_2 - \left(x_3 - \tfrac{1}{2}\right)\mathbf{a}_3 = -ax_3\,\hat{\mathbf{x}} + a\left(x_3 + \tfrac{1}{2}\right)\hat{\mathbf{y}} - a\left(x_3 - \tfrac{1}{2}\right)\hat{\mathbf{z}} \quad\text{(8c)} \quad\text{O I}$$

$$\mathbf{B_{12}} = \left(x_3 + \tfrac{1}{2}\right)\mathbf{a}_1 - \left(x_3 - \tfrac{1}{2}\right)\mathbf{a}_2 - x_3\,\mathbf{a}_3 = a\left(x_3 + \tfrac{1}{2}\right)\hat{\mathbf{x}} - a\left(x_3 - \tfrac{1}{2}\right)\hat{\mathbf{y}} - ax_3\,\hat{\mathbf{z}} \quad\text{(8c)} \quad\text{O I}$$

$$\mathbf{B_{13}} = -x_3\,\mathbf{a}_1 - x_3\,\mathbf{a}_2 - x_3\,\mathbf{a}_3 = -ax_3\,\hat{\mathbf{x}} - ax_3\,\hat{\mathbf{y}} - ax_3\,\hat{\mathbf{z}} \quad\text{(8c)} \quad\text{O I}$$

$$\mathbf{B_{14}} = \left(x_3 + \tfrac{1}{2}\right)\mathbf{a}_1 + x_3\,\mathbf{a}_2 - \left(x_3 - \tfrac{1}{2}\right)\mathbf{a}_3 = a\left(x_3 + \tfrac{1}{2}\right)\hat{\mathbf{x}} + ax_3\,\hat{\mathbf{y}} - a\left(x_3 - \tfrac{1}{2}\right)\hat{\mathbf{z}} \quad\text{(8c)} \quad\text{O I}$$

$$\mathbf{B_{15}} = x_3\,\mathbf{a}_1 - \left(x_3 - \tfrac{1}{2}\right)\mathbf{a}_2 + \left(x_3 + \tfrac{1}{2}\right)\mathbf{a}_3 = ax_3\,\hat{\mathbf{x}} - a\left(x_3 - \tfrac{1}{2}\right)\hat{\mathbf{y}} + a\left(x_3 + \tfrac{1}{2}\right)\hat{\mathbf{z}} \quad\text{(8c)} \quad\text{O I}$$

$$\mathbf{B_{16}} = -\left(x_3 - \tfrac{1}{2}\right)\mathbf{a}_1 + \left(x_3 + \tfrac{1}{2}\right)\mathbf{a}_2 + x_3\,\mathbf{a}_3 = -a\left(x_3 - \tfrac{1}{2}\right)\hat{\mathbf{x}} + a\left(x_3 + \tfrac{1}{2}\right)\hat{\mathbf{y}} + ax_3\,\hat{\mathbf{z}} \quad\text{(8c)} \quad\text{O I}$$

$$\mathbf{B_{17}} = x_4\,\mathbf{a}_1 + x_4\,\mathbf{a}_2 + x_4\,\mathbf{a}_3 = ax_4\,\hat{\mathbf{x}} + ax_4\,\hat{\mathbf{y}} + ax_4\,\hat{\mathbf{z}} \quad\text{(8c)} \quad\text{O II}$$

$$\mathbf{B_{18}} = -\left(x_4 - \tfrac{1}{2}\right)\mathbf{a}_1 - x_4\,\mathbf{a}_2 + \left(x_4 + \tfrac{1}{2}\right)\mathbf{a}_3 = -a\left(x_4 - \tfrac{1}{2}\right)\hat{\mathbf{x}} - ax_4\,\hat{\mathbf{y}} + a\left(x_4 + \tfrac{1}{2}\right)\hat{\mathbf{z}} \quad\text{(8c)} \quad\text{O II}$$

$$\mathbf{B_{19}} = -x_4\,\mathbf{a}_1 + \left(x_4 + \tfrac{1}{2}\right)\mathbf{a}_2 - \left(x_4 - \tfrac{1}{2}\right)\mathbf{a}_3 = -ax_4\,\hat{\mathbf{x}} + a\left(x_4 + \tfrac{1}{2}\right)\hat{\mathbf{y}} - a\left(x_4 - \tfrac{1}{2}\right)\hat{\mathbf{z}} \quad\text{(8c)} \quad\text{O II}$$

$$\mathbf{B_{20}} = \left(x_4 + \tfrac{1}{2}\right)\mathbf{a}_1 - \left(x_4 - \tfrac{1}{2}\right)\mathbf{a}_2 - x_4\,\mathbf{a}_3 = a\left(x_4 + \tfrac{1}{2}\right)\hat{\mathbf{x}} - a\left(x_4 - \tfrac{1}{2}\right)\hat{\mathbf{y}} - ax_4\,\hat{\mathbf{z}} \quad\text{(8c)} \quad\text{O II}$$

$$\mathbf{B_{21}} = -x_4\,\mathbf{a}_1 - x_4\,\mathbf{a}_2 - x_4\,\mathbf{a}_3 = -ax_4\,\hat{\mathbf{x}} - ax_4\,\hat{\mathbf{y}} - ax_4\,\hat{\mathbf{z}} \quad\text{(8c)} \quad\text{O II}$$

$$\mathbf{B_{22}} = \left(x_4 + \tfrac{1}{2}\right)\mathbf{a}_1 + x_4\,\mathbf{a}_2 - \left(x_4 - \tfrac{1}{2}\right)\mathbf{a}_3 = a\left(x_4 + \tfrac{1}{2}\right)\hat{\mathbf{x}} + ax_4\,\hat{\mathbf{y}} - a\left(x_4 - \tfrac{1}{2}\right)\hat{\mathbf{z}} \quad\text{(8c)} \quad\text{O II}$$

$$\mathbf{B_{23}} = x_4\,\mathbf{a}_1 - \left(x_4 - \tfrac{1}{2}\right)\mathbf{a}_2 + \left(x_4 + \tfrac{1}{2}\right)\mathbf{a}_3 = ax_4\,\hat{\mathbf{x}} - a\left(x_4 - \tfrac{1}{2}\right)\hat{\mathbf{y}} + a\left(x_4 + \tfrac{1}{2}\right)\hat{\mathbf{z}} \quad\text{(8c)} \quad\text{O II}$$

$$\mathbf{B_{24}} = -\left(x_4 - \tfrac{1}{2}\right)\mathbf{a}_1 + \left(x_4 + \tfrac{1}{2}\right)\mathbf{a}_2 + x_4\,\mathbf{a}_3 = -a\left(x_4 - \tfrac{1}{2}\right)\hat{\mathbf{x}} + a\left(x_4 + \tfrac{1}{2}\right)\hat{\mathbf{y}} + ax_4\,\hat{\mathbf{z}} \quad\text{(8c)} \quad\text{O II}$$

$$\mathbf{B_{25}} = x_5\,\mathbf{a}_1 + x_5\,\mathbf{a}_2 + x_5\,\mathbf{a}_3 = ax_5\,\hat{\mathbf{x}} + ax_5\,\hat{\mathbf{y}} + ax_5\,\hat{\mathbf{z}} \quad\text{(8c)} \quad\text{S I}$$

$$\mathbf{B_{26}} = -\left(x_5 - \tfrac{1}{2}\right)\mathbf{a}_1 - x_5\,\mathbf{a}_2 + \left(x_5 + \tfrac{1}{2}\right)\mathbf{a}_3 = -a\left(x_5 - \tfrac{1}{2}\right)\hat{\mathbf{x}} - ax_5\,\hat{\mathbf{y}} + a\left(x_5 + \tfrac{1}{2}\right)\hat{\mathbf{z}} \quad\text{(8c)} \quad\text{S I}$$

$$\mathbf{B_{27}} = -x_5\,\mathbf{a}_1 + \left(x_5 + \tfrac{1}{2}\right)\mathbf{a}_2 - \left(x_5 - \tfrac{1}{2}\right)\mathbf{a}_3 = -ax_5\,\hat{\mathbf{x}} + a\left(x_5 + \tfrac{1}{2}\right)\hat{\mathbf{y}} - a\left(x_5 - \tfrac{1}{2}\right)\hat{\mathbf{z}} \quad\text{(8c)} \quad\text{S I}$$

$$\mathbf{B_{28}} = \left(x_5 + \tfrac{1}{2}\right)\mathbf{a}_1 - \left(x_5 - \tfrac{1}{2}\right)\mathbf{a}_2 - x_5\,\mathbf{a}_3 = a\left(x_5 + \tfrac{1}{2}\right)\hat{\mathbf{x}} - a\left(x_5 - \tfrac{1}{2}\right)\hat{\mathbf{y}} - ax_5\,\hat{\mathbf{z}} \quad\text{(8c)} \quad\text{S I}$$

$$\mathbf{B_{29}} = -x_5\,\mathbf{a}_1 - x_5\,\mathbf{a}_2 - x_5\,\mathbf{a}_3 = -ax_5\,\hat{\mathbf{x}} - ax_5\,\hat{\mathbf{y}} - ax_5\,\hat{\mathbf{z}} \quad\text{(8c)} \quad\text{S I}$$

$$\mathbf{B_{30}} = \left(x_5 + \tfrac{1}{2}\right)\mathbf{a}_1 + x_5\,\mathbf{a}_2 - \left(x_5 - \tfrac{1}{2}\right)\mathbf{a}_3 = a\left(x_5 + \tfrac{1}{2}\right)\hat{\mathbf{x}} + ax_5\,\hat{\mathbf{y}} - a\left(x_5 - \tfrac{1}{2}\right)\hat{\mathbf{z}} \quad\text{(8c)} \quad\text{S I}$$

$$\mathbf{B_{31}} = x_5\,\mathbf{a}_1 - \left(x_5 - \tfrac{1}{2}\right)\mathbf{a}_2 + \left(x_5 + \tfrac{1}{2}\right)\mathbf{a}_3 = ax_5\,\hat{\mathbf{x}} - a\left(x_5 - \tfrac{1}{2}\right)\hat{\mathbf{y}} + a\left(x_5 + \tfrac{1}{2}\right)\hat{\mathbf{z}} \quad\text{(8c)} \quad\text{S I}$$

$$\mathbf{B_{32}} = -\left(x_5 - \tfrac{1}{2}\right)\mathbf{a}_1 + \left(x_5 + \tfrac{1}{2}\right)\mathbf{a}_2 + x_5\,\mathbf{a}_3 = -a\left(x_5 - \tfrac{1}{2}\right)\hat{\mathbf{x}} + a\left(x_5 + \tfrac{1}{2}\right)\hat{\mathbf{y}} + ax_5\,\hat{\mathbf{z}} \quad\text{(8c)} \quad\text{S I}$$

$$\mathbf{B_{33}} = x_6\,\mathbf{a}_1 + y_6\,\mathbf{a}_2 + z_6\,\mathbf{a}_3 = ax_6\,\hat{\mathbf{x}} + ay_6\,\hat{\mathbf{y}} + az_6\,\hat{\mathbf{z}} \quad\text{(24d)} \quad\text{H I}$$

$$\mathbf{B_{34}} = -\left(x_6 - \tfrac{1}{2}\right)\mathbf{a}_1 - y_6\,\mathbf{a}_2 + \left(z_6 + \tfrac{1}{2}\right)\mathbf{a}_3 = -a\left(x_6 - \tfrac{1}{2}\right)\hat{\mathbf{x}} - ay_6\,\hat{\mathbf{y}} + a\left(z_6 + \tfrac{1}{2}\right)\hat{\mathbf{z}} \quad\text{(24d)} \quad\text{H I}$$

$$\mathbf{B_{35}} = -x_6\,\mathbf{a}_1 + \left(y_6 + \tfrac{1}{2}\right)\mathbf{a}_2 - \left(z_6 - \tfrac{1}{2}\right)\mathbf{a}_3 = -ax_6\,\hat{\mathbf{x}} + a\left(y_6 + \tfrac{1}{2}\right)\hat{\mathbf{y}} - a\left(z_6 - \tfrac{1}{2}\right)\hat{\mathbf{z}} \quad\text{(24d)} \quad\text{H I}$$

$$\mathbf{B_{36}} = \left(x_6 + \tfrac{1}{2}\right)\mathbf{a}_1 - \left(y_6 - \tfrac{1}{2}\right)\mathbf{a}_2 - z_6\,\mathbf{a}_3 = a\left(x_6 + \tfrac{1}{2}\right)\hat{\mathbf{x}} - a\left(y_6 - \tfrac{1}{2}\right)\hat{\mathbf{y}} - az_6\,\hat{\mathbf{z}} \quad\text{(24d)} \quad\text{H I}$$

$$\mathbf{B_{37}} = z_6\,\mathbf{a}_1 + x_6\,\mathbf{a}_2 + y_6\,\mathbf{a}_3 = az_6\,\hat{\mathbf{x}} + ax_6\,\hat{\mathbf{y}} + ay_6\,\hat{\mathbf{z}} \quad\text{(24d)} \quad\text{H I}$$

$$\mathbf{B_{38}} = \left(z_6 + \tfrac{1}{2}\right)\mathbf{a}_1 - \left(x_6 - \tfrac{1}{2}\right)\mathbf{a}_2 - y_6\,\mathbf{a}_3 = a\left(z_6 + \tfrac{1}{2}\right)\hat{\mathbf{x}} - a\left(x_6 - \tfrac{1}{2}\right)\hat{\mathbf{y}} - ay_6\,\hat{\mathbf{z}} \quad\text{(24d)} \quad\text{H I}$$

$$\mathbf{B_{39}} = -\left(z_6 - \tfrac{1}{2}\right)\mathbf{a}_1 - x_6\,\mathbf{a}_2 + \left(y_6 + \tfrac{1}{2}\right)\mathbf{a}_3 = -a\left(z_6 - \tfrac{1}{2}\right)\hat{\mathbf{x}} - ax_6\,\hat{\mathbf{y}} + a\left(y_6 + \tfrac{1}{2}\right)\hat{\mathbf{z}} \qquad (24\text{d}) \qquad \text{H I}$$

$$\mathbf{B_{40}} = -z_6\,\mathbf{a}_1 + \left(x_6 + \tfrac{1}{2}\right)\mathbf{a}_2 - \left(y_6 - \tfrac{1}{2}\right)\mathbf{a}_3 = -az_6\,\hat{\mathbf{x}} + a\left(x_6 + \tfrac{1}{2}\right)\hat{\mathbf{y}} - a\left(y_6 - \tfrac{1}{2}\right)\hat{\mathbf{z}} \qquad (24\text{d}) \qquad \text{H I}$$

$$\mathbf{B_{41}} = y_6\,\mathbf{a}_1 + z_6\,\mathbf{a}_2 + x_6\,\mathbf{a}_3 = ay_6\,\hat{\mathbf{x}} + az_6\,\hat{\mathbf{y}} + ax_6\,\hat{\mathbf{z}} \qquad (24\text{d}) \qquad \text{H I}$$

$$\mathbf{B_{42}} = -y_6\,\mathbf{a}_1 + \left(z_6 + \tfrac{1}{2}\right)\mathbf{a}_2 - \left(x_6 - \tfrac{1}{2}\right)\mathbf{a}_3 = -ay_6\,\hat{\mathbf{x}} + a\left(z_6 + \tfrac{1}{2}\right)\hat{\mathbf{y}} - a\left(x_6 - \tfrac{1}{2}\right)\hat{\mathbf{z}} \qquad (24\text{d}) \qquad \text{H I}$$

$$\mathbf{B_{43}} = \left(y_6 + \tfrac{1}{2}\right)\mathbf{a}_1 - \left(z_6 - \tfrac{1}{2}\right)\mathbf{a}_2 - x_6\,\mathbf{a}_3 = a\left(y_6 + \tfrac{1}{2}\right)\hat{\mathbf{x}} - a\left(z_6 - \tfrac{1}{2}\right)\hat{\mathbf{y}} - ax_6\,\hat{\mathbf{z}} \qquad (24\text{d}) \qquad \text{H I}$$

$$\mathbf{B_{44}} = -\left(y_6 - \tfrac{1}{2}\right)\mathbf{a}_1 - z_6\,\mathbf{a}_2 + \left(x_6 + \tfrac{1}{2}\right)\mathbf{a}_3 = -a\left(y_6 - \tfrac{1}{2}\right)\hat{\mathbf{x}} - az_6\,\hat{\mathbf{y}} + a\left(x_6 + \tfrac{1}{2}\right)\hat{\mathbf{z}} \qquad (24\text{d}) \qquad \text{H I}$$

$$\mathbf{B_{45}} = -x_6\,\mathbf{a}_1 - y_6\,\mathbf{a}_2 - z_6\,\mathbf{a}_3 = -ax_6\,\hat{\mathbf{x}} - ay_6\,\hat{\mathbf{y}} - az_6\,\hat{\mathbf{z}} \qquad (24\text{d}) \qquad \text{H I}$$

$$\mathbf{B_{46}} = \left(x_6 + \tfrac{1}{2}\right)\mathbf{a}_1 + y_6\,\mathbf{a}_2 - \left(z_6 - \tfrac{1}{2}\right)\mathbf{a}_3 = a\left(x_6 + \tfrac{1}{2}\right)\hat{\mathbf{x}} + ay_6\,\hat{\mathbf{y}} - a\left(z_6 - \tfrac{1}{2}\right)\hat{\mathbf{z}} \qquad (24\text{d}) \qquad \text{H I}$$

$$\mathbf{B_{47}} = x_6\,\mathbf{a}_1 - \left(y_6 - \tfrac{1}{2}\right)\mathbf{a}_2 + \left(z_6 + \tfrac{1}{2}\right)\mathbf{a}_3 = ax_6\,\hat{\mathbf{x}} - a\left(y_6 - \tfrac{1}{2}\right)\hat{\mathbf{y}} + a\left(z_6 + \tfrac{1}{2}\right)\hat{\mathbf{z}} \qquad (24\text{d}) \qquad \text{H I}$$

$$\mathbf{B_{48}} = -\left(x_6 - \tfrac{1}{2}\right)\mathbf{a}_1 + \left(y_6 + \tfrac{1}{2}\right)\mathbf{a}_2 + z_6\,\mathbf{a}_3 = -a\left(x_6 - \tfrac{1}{2}\right)\hat{\mathbf{x}} + a\left(y_6 + \tfrac{1}{2}\right)\hat{\mathbf{y}} + az_6\,\hat{\mathbf{z}} \qquad (24\text{d}) \qquad \text{H I}$$

$$\mathbf{B_{49}} = -z_6\,\mathbf{a}_1 - x_6\,\mathbf{a}_2 - y_6\,\mathbf{a}_3 = -az_6\,\hat{\mathbf{x}} - ax_6\,\hat{\mathbf{y}} - ay_6\,\hat{\mathbf{z}} \qquad (24\text{d}) \qquad \text{H I}$$

$$\mathbf{B_{50}} = -\left(z_6 - \tfrac{1}{2}\right)\mathbf{a}_1 + \left(x_6 + \tfrac{1}{2}\right)\mathbf{a}_2 + y_6\,\mathbf{a}_3 = -a\left(z_6 - \tfrac{1}{2}\right)\hat{\mathbf{x}} + a\left(x_6 + \tfrac{1}{2}\right)\hat{\mathbf{y}} + ay_6\,\hat{\mathbf{z}} \qquad (24\text{d}) \qquad \text{H I}$$

$$\mathbf{B_{51}} = \left(z_6 + \tfrac{1}{2}\right)\mathbf{a}_1 + x_6\,\mathbf{a}_2 - \left(y_6 - \tfrac{1}{2}\right)\mathbf{a}_3 = a\left(z_6 + \tfrac{1}{2}\right)\hat{\mathbf{x}} + ax_6\,\hat{\mathbf{y}} - a\left(y_6 - \tfrac{1}{2}\right)\hat{\mathbf{z}} \qquad (24\text{d}) \qquad \text{H I}$$

$$\mathbf{B_{52}} = z_6\,\mathbf{a}_1 - \left(x_6 - \tfrac{1}{2}\right)\mathbf{a}_2 + \left(y_6 + \tfrac{1}{2}\right)\mathbf{a}_3 = az_6\,\hat{\mathbf{x}} - a\left(x_6 - \tfrac{1}{2}\right)\hat{\mathbf{y}} + a\left(y_6 + \tfrac{1}{2}\right)\hat{\mathbf{z}} \qquad (24\text{d}) \qquad \text{H I}$$

$$\mathbf{B_{53}} = -y_6\,\mathbf{a}_1 - z_6\,\mathbf{a}_2 - x_6\,\mathbf{a}_3 = -ay_6\,\hat{\mathbf{x}} - az_6\,\hat{\mathbf{y}} - ax_6\,\hat{\mathbf{z}} \qquad (24\text{d}) \qquad \text{H I}$$

$$\mathbf{B_{54}} = y_6\,\mathbf{a}_1 - \left(z_6 - \tfrac{1}{2}\right)\mathbf{a}_2 + \left(x_6 + \tfrac{1}{2}\right)\mathbf{a}_3 = ay_6\,\hat{\mathbf{x}} - a\left(z_6 - \tfrac{1}{2}\right)\hat{\mathbf{y}} + a\left(x_6 + \tfrac{1}{2}\right)\hat{\mathbf{z}} \qquad (24\text{d}) \qquad \text{H I}$$

$$\mathbf{B_{55}} = -\left(y_6 - \tfrac{1}{2}\right)\mathbf{a}_1 + \left(z_6 + \tfrac{1}{2}\right)\mathbf{a}_2 + x_6\,\mathbf{a}_3 = -a\left(y_6 - \tfrac{1}{2}\right)\hat{\mathbf{x}} + a\left(z_6 + \tfrac{1}{2}\right)\hat{\mathbf{y}} + ax_6\,\hat{\mathbf{z}} \qquad (24\text{d}) \qquad \text{H I}$$

$$\mathbf{B_{56}} = \left(y_6 + \tfrac{1}{2}\right)\mathbf{a}_1 + z_6\,\mathbf{a}_2 - \left(x_6 - \tfrac{1}{2}\right)\mathbf{a}_3 = a\left(y_6 + \tfrac{1}{2}\right)\hat{\mathbf{x}} + az_6\,\hat{\mathbf{y}} - a\left(x_6 - \tfrac{1}{2}\right)\hat{\mathbf{z}} \qquad (24\text{d}) \qquad \text{H I}$$

$$\mathbf{B_{57}} = x_7\,\mathbf{a}_1 + y_7\,\mathbf{a}_2 + z_7\,\mathbf{a}_3 = ax_7\,\hat{\mathbf{x}} + ay_7\,\hat{\mathbf{y}} + az_7\,\hat{\mathbf{z}} \qquad (24\text{d}) \qquad \text{H II}$$

$$\mathbf{B_{58}} = -\left(x_7 - \tfrac{1}{2}\right)\mathbf{a}_1 - y_7\,\mathbf{a}_2 + \left(z_7 + \tfrac{1}{2}\right)\mathbf{a}_3 = -a\left(x_7 - \tfrac{1}{2}\right)\hat{\mathbf{x}} - ay_7\,\hat{\mathbf{y}} + a\left(z_7 + \tfrac{1}{2}\right)\hat{\mathbf{z}} \qquad (24\text{d}) \qquad \text{H II}$$

$$\mathbf{B_{59}} = -x_7\,\mathbf{a}_1 + \left(y_7 + \tfrac{1}{2}\right)\mathbf{a}_2 - \left(z_7 - \tfrac{1}{2}\right)\mathbf{a}_3 = -ax_7\,\hat{\mathbf{x}} + a\left(y_7 + \tfrac{1}{2}\right)\hat{\mathbf{y}} - a\left(z_7 - \tfrac{1}{2}\right)\hat{\mathbf{z}} \qquad (24\text{d}) \qquad \text{H II}$$

$$\mathbf{B_{60}} = \left(x_7 + \tfrac{1}{2}\right)\mathbf{a}_1 - \left(y_7 - \tfrac{1}{2}\right)\mathbf{a}_2 - z_7\,\mathbf{a}_3 = a\left(x_7 + \tfrac{1}{2}\right)\hat{\mathbf{x}} - a\left(y_7 - \tfrac{1}{2}\right)\hat{\mathbf{y}} - az_7\,\hat{\mathbf{z}} \qquad (24\text{d}) \qquad \text{H II}$$

$$\mathbf{B_{61}} = z_7\,\mathbf{a}_1 + x_7\,\mathbf{a}_2 + y_7\,\mathbf{a}_3 = az_7\,\hat{\mathbf{x}} + ax_7\,\hat{\mathbf{y}} + ay_7\,\hat{\mathbf{z}} \qquad (24\text{d}) \qquad \text{H II}$$

$$\mathbf{B_{62}} = \left(z_7 + \tfrac{1}{2}\right)\mathbf{a}_1 - \left(x_7 - \tfrac{1}{2}\right)\mathbf{a}_2 - y_7\,\mathbf{a}_3 = a\left(z_7 + \tfrac{1}{2}\right)\hat{\mathbf{x}} - a\left(x_7 - \tfrac{1}{2}\right)\hat{\mathbf{y}} - ay_7\,\hat{\mathbf{z}} \qquad (24\text{d}) \qquad \text{H II}$$

$$\mathbf{B_{63}} = -\left(z_7 - \tfrac{1}{2}\right)\mathbf{a}_1 - x_7\,\mathbf{a}_2 + \left(y_7 + \tfrac{1}{2}\right)\mathbf{a}_3 = -a\left(z_7 - \tfrac{1}{2}\right)\hat{\mathbf{x}} - ax_7\,\hat{\mathbf{y}} + a\left(y_7 + \tfrac{1}{2}\right)\hat{\mathbf{z}} \qquad (24\text{d}) \qquad \text{H II}$$

$$\mathbf{B_{64}} = -z_7\,\mathbf{a}_1 + \left(x_7 + \tfrac{1}{2}\right)\mathbf{a}_2 - \left(y_7 - \tfrac{1}{2}\right)\mathbf{a}_3 = -az_7\,\hat{\mathbf{x}} + a\left(x_7 + \tfrac{1}{2}\right)\hat{\mathbf{y}} - a\left(y_7 - \tfrac{1}{2}\right)\hat{\mathbf{z}} \qquad (24\text{d}) \qquad \text{H II}$$

$$\mathbf{B_{65}} = y_7\,\mathbf{a}_1 + z_7\,\mathbf{a}_2 + x_7\,\mathbf{a}_3 = ay_7\,\hat{\mathbf{x}} + az_7\,\hat{\mathbf{y}} + ax_7\,\hat{\mathbf{z}} \qquad (24\text{d}) \qquad \text{H II}$$

$$\mathbf{B_{66}} = -y_7\,\mathbf{a}_1 + \left(z_7 + \tfrac{1}{2}\right)\mathbf{a}_2 - \left(x_7 - \tfrac{1}{2}\right)\mathbf{a}_3 = -ay_7\,\hat{\mathbf{x}} + a\left(z_7 + \tfrac{1}{2}\right)\hat{\mathbf{y}} - a\left(x_7 - \tfrac{1}{2}\right)\hat{\mathbf{z}} \qquad (24\text{d}) \qquad \text{H II}$$

$$\mathbf{B_{67}} = \left(y_7 + \tfrac{1}{2}\right)\mathbf{a}_1 - \left(z_7 - \tfrac{1}{2}\right)\mathbf{a}_2 - x_7\,\mathbf{a}_3 = a\left(y_7 + \tfrac{1}{2}\right)\hat{\mathbf{x}} - a\left(z_7 - \tfrac{1}{2}\right)\hat{\mathbf{y}} - ax_7\,\hat{\mathbf{z}} \qquad (24\text{d}) \qquad \text{H II}$$

$$\mathbf{B_{68}} = -\left(y_7 - \tfrac{1}{2}\right)\mathbf{a}_1 - z_7\,\mathbf{a}_2 + \left(x_7 + \tfrac{1}{2}\right)\mathbf{a}_3 = -a\left(y_7 - \tfrac{1}{2}\right)\hat{\mathbf{x}} - az_7\,\hat{\mathbf{y}} + a\left(x_7 + \tfrac{1}{2}\right)\hat{\mathbf{z}} \qquad (24\text{d}) \qquad \text{H II}$$

$$\mathbf{B_{69}} = -x_7\,\mathbf{a}_1 - y_7\,\mathbf{a}_2 - z_7\,\mathbf{a}_3 = -ax_7\,\hat{\mathbf{x}} - ay_7\,\hat{\mathbf{y}} - az_7\,\hat{\mathbf{z}} \qquad (24\text{d}) \qquad \text{H II}$$

$$
\begin{array}{llllll}
\mathbf{B_{70}} & = & \left(x_7 + \tfrac{1}{2}\right)\mathbf{a}_1 + y_7\,\mathbf{a}_2 - \left(z_7 - \tfrac{1}{2}\right)\mathbf{a}_3 & = & a\left(x_7 + \tfrac{1}{2}\right)\hat{\mathbf{x}} + ay_7\,\hat{\mathbf{y}} - a\left(z_7 - \tfrac{1}{2}\right)\hat{\mathbf{z}} & \text{(24d)} \qquad \text{H II} \\
\mathbf{B_{71}} & = & x_7\,\mathbf{a}_1 - \left(y_7 - \tfrac{1}{2}\right)\mathbf{a}_2 + \left(z_7 + \tfrac{1}{2}\right)\mathbf{a}_3 & = & ax_7\,\hat{\mathbf{x}} - a\left(y_7 - \tfrac{1}{2}\right)\hat{\mathbf{y}} + a\left(z_7 + \tfrac{1}{2}\right)\hat{\mathbf{z}} & \text{(24d)} \qquad \text{H II} \\
\mathbf{B_{72}} & = & -\left(x_7 - \tfrac{1}{2}\right)\mathbf{a}_1 + \left(y_7 + \tfrac{1}{2}\right)\mathbf{a}_2 + z_7\,\mathbf{a}_3 & = & -a\left(x_7 - \tfrac{1}{2}\right)\hat{\mathbf{x}} + a\left(y_7 + \tfrac{1}{2}\right)\hat{\mathbf{y}} + az_7\,\hat{\mathbf{z}} & \text{(24d)} \qquad \text{H II} \\
\mathbf{B_{73}} & = & -z_7\,\mathbf{a}_1 - x_7\,\mathbf{a}_2 - y_7\,\mathbf{a}_3 & = & -az_7\,\hat{\mathbf{x}} - ax_7\,\hat{\mathbf{y}} - ay_7\,\hat{\mathbf{z}} & \text{(24d)} \qquad \text{H II} \\
\mathbf{B_{74}} & = & -\left(z_7 - \tfrac{1}{2}\right)\mathbf{a}_1 + \left(x_7 + \tfrac{1}{2}\right)\mathbf{a}_2 + y_7\,\mathbf{a}_3 & = & -a\left(z_7 - \tfrac{1}{2}\right)\hat{\mathbf{x}} + a\left(x_7 + \tfrac{1}{2}\right)\hat{\mathbf{y}} + ay_7\,\hat{\mathbf{z}} & \text{(24d)} \qquad \text{H II} \\
\mathbf{B_{75}} & = & \left(z_7 + \tfrac{1}{2}\right)\mathbf{a}_1 + x_7\,\mathbf{a}_2 - \left(y_7 - \tfrac{1}{2}\right)\mathbf{a}_3 & = & a\left(z_7 + \tfrac{1}{2}\right)\hat{\mathbf{x}} + ax_7\,\hat{\mathbf{y}} - a\left(y_7 - \tfrac{1}{2}\right)\hat{\mathbf{z}} & \text{(24d)} \qquad \text{H II} \\
\mathbf{B_{76}} & = & z_7\,\mathbf{a}_1 - \left(x_7 - \tfrac{1}{2}\right)\mathbf{a}_2 + \left(y_7 + \tfrac{1}{2}\right)\mathbf{a}_3 & = & az_7\,\hat{\mathbf{x}} - a\left(x_7 - \tfrac{1}{2}\right)\hat{\mathbf{y}} + a\left(y_7 + \tfrac{1}{2}\right)\hat{\mathbf{z}} & \text{(24d)} \qquad \text{H II} \\
\mathbf{B_{77}} & = & -y_7\,\mathbf{a}_1 - z_7\,\mathbf{a}_2 - x_7\,\mathbf{a}_3 & = & -ay_7\,\hat{\mathbf{x}} - az_7\,\hat{\mathbf{y}} - ax_7\,\hat{\mathbf{z}} & \text{(24d)} \qquad \text{H II} \\
\mathbf{B_{78}} & = & y_7\,\mathbf{a}_1 - \left(z_7 - \tfrac{1}{2}\right)\mathbf{a}_2 + \left(x_7 + \tfrac{1}{2}\right)\mathbf{a}_3 & = & ay_7\,\hat{\mathbf{x}} - a\left(z_7 - \tfrac{1}{2}\right)\hat{\mathbf{y}} + a\left(x_7 + \tfrac{1}{2}\right)\hat{\mathbf{z}} & \text{(24d)} \qquad \text{H II} \\
\mathbf{B_{79}} & = & -\left(y_7 - \tfrac{1}{2}\right)\mathbf{a}_1 + \left(z_7 + \tfrac{1}{2}\right)\mathbf{a}_2 + x_7\,\mathbf{a}_3 & = & -a\left(y_7 - \tfrac{1}{2}\right)\hat{\mathbf{x}} + a\left(z_7 + \tfrac{1}{2}\right)\hat{\mathbf{y}} + ax_7\,\hat{\mathbf{z}} & \text{(24d)} \qquad \text{H II} \\
\mathbf{B_{80}} & = & \left(y_7 + \tfrac{1}{2}\right)\mathbf{a}_1 + z_7\,\mathbf{a}_2 - \left(x_7 - \tfrac{1}{2}\right)\mathbf{a}_3 & = & a\left(y_7 + \tfrac{1}{2}\right)\hat{\mathbf{x}} + az_7\,\hat{\mathbf{y}} - a\left(x_7 - \tfrac{1}{2}\right)\hat{\mathbf{z}} & \text{(24d)} \qquad \text{H II} \\
\mathbf{B_{81}} & = & x_8\,\mathbf{a}_1 + y_8\,\mathbf{a}_2 + z_8\,\mathbf{a}_3 & = & ax_8\,\hat{\mathbf{x}} + ay_8\,\hat{\mathbf{y}} + az_8\,\hat{\mathbf{z}} & \text{(24d)} \qquad \text{H III} \\
\mathbf{B_{82}} & = & -\left(x_8 - \tfrac{1}{2}\right)\mathbf{a}_1 - y_8\,\mathbf{a}_2 + \left(z_8 + \tfrac{1}{2}\right)\mathbf{a}_3 & = & -a\left(x_8 - \tfrac{1}{2}\right)\hat{\mathbf{x}} - ay_8\,\hat{\mathbf{y}} + a\left(z_8 + \tfrac{1}{2}\right)\hat{\mathbf{z}} & \text{(24d)} \qquad \text{H III} \\
\mathbf{B_{83}} & = & -x_8\,\mathbf{a}_1 + \left(y_8 + \tfrac{1}{2}\right)\mathbf{a}_2 - \left(z_8 - \tfrac{1}{2}\right)\mathbf{a}_3 & = & -ax_8\,\hat{\mathbf{x}} + a\left(y_8 + \tfrac{1}{2}\right)\hat{\mathbf{y}} - a\left(z_8 - \tfrac{1}{2}\right)\hat{\mathbf{z}} & \text{(24d)} \qquad \text{H III} \\
\mathbf{B_{84}} & = & \left(x_8 + \tfrac{1}{2}\right)\mathbf{a}_1 - \left(y_8 - \tfrac{1}{2}\right)\mathbf{a}_2 - z_8\,\mathbf{a}_3 & = & a\left(x_8 + \tfrac{1}{2}\right)\hat{\mathbf{x}} - a\left(y_8 - \tfrac{1}{2}\right)\hat{\mathbf{y}} - az_8\,\hat{\mathbf{z}} & \text{(24d)} \qquad \text{H III} \\
\mathbf{B_{85}} & = & z_8\,\mathbf{a}_1 + x_8\,\mathbf{a}_2 + y_8\,\mathbf{a}_3 & = & az_8\,\hat{\mathbf{x}} + ax_8\,\hat{\mathbf{y}} + ay_8\,\hat{\mathbf{z}} & \text{(24d)} \qquad \text{H III} \\
\mathbf{B_{86}} & = & \left(z_8 + \tfrac{1}{2}\right)\mathbf{a}_1 - \left(x_8 - \tfrac{1}{2}\right)\mathbf{a}_2 - y_8\,\mathbf{a}_3 & = & a\left(z_8 + \tfrac{1}{2}\right)\hat{\mathbf{x}} - a\left(x_8 - \tfrac{1}{2}\right)\hat{\mathbf{y}} - ay_8\,\hat{\mathbf{z}} & \text{(24d)} \qquad \text{H III} \\
\mathbf{B_{87}} & = & -\left(z_8 - \tfrac{1}{2}\right)\mathbf{a}_1 - x_8\,\mathbf{a}_2 + \left(y_8 + \tfrac{1}{2}\right)\mathbf{a}_3 & = & -a\left(z_8 - \tfrac{1}{2}\right)\hat{\mathbf{x}} - ax_8\,\hat{\mathbf{y}} + a\left(y_8 + \tfrac{1}{2}\right)\hat{\mathbf{z}} & \text{(24d)} \qquad \text{H III} \\
\mathbf{B_{88}} & = & -z_8\,\mathbf{a}_1 + \left(x_8 + \tfrac{1}{2}\right)\mathbf{a}_2 - \left(y_8 - \tfrac{1}{2}\right)\mathbf{a}_3 & = & -az_8\,\hat{\mathbf{x}} + a\left(x_8 + \tfrac{1}{2}\right)\hat{\mathbf{y}} - a\left(y_8 - \tfrac{1}{2}\right)\hat{\mathbf{z}} & \text{(24d)} \qquad \text{H III} \\
\mathbf{B_{89}} & = & y_8\,\mathbf{a}_1 + z_8\,\mathbf{a}_2 + x_8\,\mathbf{a}_3 & = & ay_8\,\hat{\mathbf{x}} + az_8\,\hat{\mathbf{y}} + ax_8\,\hat{\mathbf{z}} & \text{(24d)} \qquad \text{H III} \\
\mathbf{B_{90}} & = & -y_8\,\mathbf{a}_1 + \left(z_8 + \tfrac{1}{2}\right)\mathbf{a}_2 - \left(x_8 - \tfrac{1}{2}\right)\mathbf{a}_3 & = & -ay_8\,\hat{\mathbf{x}} + a\left(z_8 + \tfrac{1}{2}\right)\hat{\mathbf{y}} - a\left(x_8 - \tfrac{1}{2}\right)\hat{\mathbf{z}} & \text{(24d)} \qquad \text{H III} \\
\mathbf{B_{91}} & = & \left(y_8 + \tfrac{1}{2}\right)\mathbf{a}_1 - \left(z_8 - \tfrac{1}{2}\right)\mathbf{a}_2 - x_8\,\mathbf{a}_3 & = & a\left(y_8 + \tfrac{1}{2}\right)\hat{\mathbf{x}} - a\left(z_8 - \tfrac{1}{2}\right)\hat{\mathbf{y}} - ax_8\,\hat{\mathbf{z}} & \text{(24d)} \qquad \text{H III} \\
\mathbf{B_{92}} & = & -\left(y_8 - \tfrac{1}{2}\right)\mathbf{a}_1 - z_8\,\mathbf{a}_2 + \left(x_8 + \tfrac{1}{2}\right)\mathbf{a}_3 & = & -a\left(y_8 - \tfrac{1}{2}\right)\hat{\mathbf{x}} - az_8\,\hat{\mathbf{y}} + a\left(x_8 + \tfrac{1}{2}\right)\hat{\mathbf{z}} & \text{(24d)} \qquad \text{H III} \\
\mathbf{B_{93}} & = & -x_8\,\mathbf{a}_1 - y_8\,\mathbf{a}_2 - z_8\,\mathbf{a}_3 & = & -ax_8\,\hat{\mathbf{x}} - ay_8\,\hat{\mathbf{y}} - az_8\,\hat{\mathbf{z}} & \text{(24d)} \qquad \text{H III} \\
\mathbf{B_{94}} & = & \left(x_8 + \tfrac{1}{2}\right)\mathbf{a}_1 + y_8\,\mathbf{a}_2 - \left(z_8 - \tfrac{1}{2}\right)\mathbf{a}_3 & = & a\left(x_8 + \tfrac{1}{2}\right)\hat{\mathbf{x}} + ay_8\,\hat{\mathbf{y}} - a\left(z_8 - \tfrac{1}{2}\right)\hat{\mathbf{z}} & \text{(24d)} \qquad \text{H III} \\
\mathbf{B_{95}} & = & x_8\,\mathbf{a}_1 - \left(y_8 - \tfrac{1}{2}\right)\mathbf{a}_2 + \left(z_8 + \tfrac{1}{2}\right)\mathbf{a}_3 & = & ax_8\,\hat{\mathbf{x}} - a\left(y_8 - \tfrac{1}{2}\right)\hat{\mathbf{y}} + a\left(z_8 + \tfrac{1}{2}\right)\hat{\mathbf{z}} & \text{(24d)} \qquad \text{H III} \\
\mathbf{B_{96}} & = & -\left(x_8 - \tfrac{1}{2}\right)\mathbf{a}_1 + \left(y_8 + \tfrac{1}{2}\right)\mathbf{a}_2 + z_8\,\mathbf{a}_3 & = & -a\left(x_8 - \tfrac{1}{2}\right)\hat{\mathbf{x}} + a\left(y_8 + \tfrac{1}{2}\right)\hat{\mathbf{y}} + az_8\,\hat{\mathbf{z}} & \text{(24d)} \qquad \text{H III} \\
\mathbf{B_{97}} & = & -z_8\,\mathbf{a}_1 - x_8\,\mathbf{a}_2 - y_8\,\mathbf{a}_3 & = & -az_8\,\hat{\mathbf{x}} - ax_8\,\hat{\mathbf{y}} - ay_8\,\hat{\mathbf{z}} & \text{(24d)} \qquad \text{H III} \\
\mathbf{B_{98}} & = & -\left(z_8 - \tfrac{1}{2}\right)\mathbf{a}_1 + \left(x_8 + \tfrac{1}{2}\right)\mathbf{a}_2 + y_8\,\mathbf{a}_3 & = & -a\left(z_8 - \tfrac{1}{2}\right)\hat{\mathbf{x}} + a\left(x_8 + \tfrac{1}{2}\right)\hat{\mathbf{y}} + ay_8\,\hat{\mathbf{z}} & \text{(24d)} \qquad \text{H III} \\
\mathbf{B_{99}} & = & \left(z_8 + \tfrac{1}{2}\right)\mathbf{a}_1 + x_8\,\mathbf{a}_2 - \left(y_8 - \tfrac{1}{2}\right)\mathbf{a}_3 & = & a\left(z_8 + \tfrac{1}{2}\right)\hat{\mathbf{x}} + ax_8\,\hat{\mathbf{y}} - a\left(y_8 - \tfrac{1}{2}\right)\hat{\mathbf{z}} & \text{(24d)} \qquad \text{H III} \\
\mathbf{B_{100}} & = & z_8\,\mathbf{a}_1 - \left(x_8 - \tfrac{1}{2}\right)\mathbf{a}_2 + \left(y_8 + \tfrac{1}{2}\right)\mathbf{a}_3 & = & az_8\,\hat{\mathbf{x}} - a\left(x_8 - \tfrac{1}{2}\right)\hat{\mathbf{y}} + a\left(y_8 + \tfrac{1}{2}\right)\hat{\mathbf{z}} & \text{(24d)} \qquad \text{H III} \\
\mathbf{B_{101}} & = & -y_8\,\mathbf{a}_1 - z_8\,\mathbf{a}_2 - x_8\,\mathbf{a}_3 & = & -ay_8\,\hat{\mathbf{x}} - az_8\,\hat{\mathbf{y}} - ax_8\,\hat{\mathbf{z}} & \text{(24d)} \qquad \text{H III} \\
\mathbf{B_{102}} & = & y_8\,\mathbf{a}_1 - \left(z_8 - \tfrac{1}{2}\right)\mathbf{a}_2 + \left(x_8 + \tfrac{1}{2}\right)\mathbf{a}_3 & = & ay_8\,\hat{\mathbf{x}} - a\left(z_8 - \tfrac{1}{2}\right)\hat{\mathbf{y}} + a\left(x_8 + \tfrac{1}{2}\right)\hat{\mathbf{z}} & \text{(24d)} \qquad \text{H III}
\end{array}
$$

$$\mathbf{B_{103}} = \quad -\left(y_8 - \tfrac{1}{2}\right)\mathbf{a}_1 + \left(z_8 + \tfrac{1}{2}\right)\mathbf{a}_2 + x_8\,\mathbf{a}_3 \quad = \quad -a\left(y_8 - \tfrac{1}{2}\right)\hat{\mathbf{x}} + a\left(z_8 + \tfrac{1}{2}\right)\hat{\mathbf{y}} + ax_8\,\hat{\mathbf{z}} \qquad (24\mathrm{d}) \qquad \mathrm{H\ III}$$

$$\mathbf{B_{104}} = \quad \left(y_8 + \tfrac{1}{2}\right)\mathbf{a}_1 + z_8\,\mathbf{a}_2 - \left(x_8 - \tfrac{1}{2}\right)\mathbf{a}_3 \quad = \quad a\left(y_8 + \tfrac{1}{2}\right)\hat{\mathbf{x}} + az_8\,\hat{\mathbf{y}} - a\left(x_8 - \tfrac{1}{2}\right)\hat{\mathbf{z}} \qquad (24\mathrm{d}) \qquad \mathrm{H\ III}$$

$$\mathbf{B_{105}} = \quad x_9\,\mathbf{a}_1 + y_9\,\mathbf{a}_2 + z_9\,\mathbf{a}_3 \quad = \quad ax_9\,\hat{\mathbf{x}} + ay_9\,\hat{\mathbf{y}} + az_9\,\hat{\mathbf{z}} \qquad (24\mathrm{d}) \qquad \mathrm{H\ IV}$$

$$\mathbf{B_{106}} = \quad -\left(x_9 - \tfrac{1}{2}\right)\mathbf{a}_1 - y_9\,\mathbf{a}_2 + \left(z_9 + \tfrac{1}{2}\right)\mathbf{a}_3 \quad = \quad -a\left(x_9 - \tfrac{1}{2}\right)\hat{\mathbf{x}} - ay_9\,\hat{\mathbf{y}} + a\left(z_9 + \tfrac{1}{2}\right)\hat{\mathbf{z}} \qquad (24\mathrm{d}) \qquad \mathrm{H\ IV}$$

$$\mathbf{B_{107}} = \quad -x_9\,\mathbf{a}_1 + \left(y_9 + \tfrac{1}{2}\right)\mathbf{a}_2 - \left(z_9 - \tfrac{1}{2}\right)\mathbf{a}_3 \quad = \quad -ax_9\,\hat{\mathbf{x}} + a\left(y_9 + \tfrac{1}{2}\right)\hat{\mathbf{y}} - a\left(z_9 - \tfrac{1}{2}\right)\hat{\mathbf{z}} \qquad (24\mathrm{d}) \qquad \mathrm{H\ IV}$$

$$\mathbf{B_{108}} = \quad \left(x_9 + \tfrac{1}{2}\right)\mathbf{a}_1 - \left(y_9 - \tfrac{1}{2}\right)\mathbf{a}_2 - z_9\,\mathbf{a}_3 \quad = \quad a\left(x_9 + \tfrac{1}{2}\right)\hat{\mathbf{x}} - a\left(y_9 - \tfrac{1}{2}\right)\hat{\mathbf{y}} - az_9\,\hat{\mathbf{z}} \qquad (24\mathrm{d}) \qquad \mathrm{H\ IV}$$

$$\mathbf{B_{109}} = \quad z_9\,\mathbf{a}_1 + x_9\,\mathbf{a}_2 + y_9\,\mathbf{a}_3 \quad = \quad az_9\,\hat{\mathbf{x}} + ax_9\,\hat{\mathbf{y}} + ay_9\,\hat{\mathbf{z}} \qquad (24\mathrm{d}) \qquad \mathrm{H\ IV}$$

$$\mathbf{B_{110}} = \quad \left(z_9 + \tfrac{1}{2}\right)\mathbf{a}_1 - \left(x_9 - \tfrac{1}{2}\right)\mathbf{a}_2 - y_9\,\mathbf{a}_3 \quad = \quad a\left(z_9 + \tfrac{1}{2}\right)\hat{\mathbf{x}} - a\left(x_9 - \tfrac{1}{2}\right)\hat{\mathbf{y}} - ay_9\,\hat{\mathbf{z}} \qquad (24\mathrm{d}) \qquad \mathrm{H\ IV}$$

$$\mathbf{B_{111}} = \quad -\left(z_9 - \tfrac{1}{2}\right)\mathbf{a}_1 - x_9\,\mathbf{a}_2 + \left(y_9 + \tfrac{1}{2}\right)\mathbf{a}_3 \quad = \quad -a\left(z_9 - \tfrac{1}{2}\right)\hat{\mathbf{x}} - ax_9\,\hat{\mathbf{y}} + a\left(y_9 + \tfrac{1}{2}\right)\hat{\mathbf{z}} \qquad (24\mathrm{d}) \qquad \mathrm{H\ IV}$$

$$\mathbf{B_{112}} = \quad -z_9\,\mathbf{a}_1 + \left(x_9 + \tfrac{1}{2}\right)\mathbf{a}_2 - \left(y_9 - \tfrac{1}{2}\right)\mathbf{a}_3 \quad = \quad -az_9\,\hat{\mathbf{x}} + a\left(x_9 + \tfrac{1}{2}\right)\hat{\mathbf{y}} - a\left(y_9 - \tfrac{1}{2}\right)\hat{\mathbf{z}} \qquad (24\mathrm{d}) \qquad \mathrm{H\ IV}$$

$$\mathbf{B_{113}} = \quad y_9\,\mathbf{a}_1 + z_9\,\mathbf{a}_2 + x_9\,\mathbf{a}_3 \quad = \quad ay_9\,\hat{\mathbf{x}} + az_9\,\hat{\mathbf{y}} + ax_9\,\hat{\mathbf{z}} \qquad (24\mathrm{d}) \qquad \mathrm{H\ IV}$$

$$\mathbf{B_{114}} = \quad -y_9\,\mathbf{a}_1 + \left(z_9 + \tfrac{1}{2}\right)\mathbf{a}_2 - \left(x_9 - \tfrac{1}{2}\right)\mathbf{a}_3 \quad = \quad -ay_9\,\hat{\mathbf{x}} + a\left(z_9 + \tfrac{1}{2}\right)\hat{\mathbf{y}} - a\left(x_9 - \tfrac{1}{2}\right)\hat{\mathbf{z}} \qquad (24\mathrm{d}) \qquad \mathrm{H\ IV}$$

$$\mathbf{B_{115}} = \quad \left(y_9 + \tfrac{1}{2}\right)\mathbf{a}_1 - \left(z_9 - \tfrac{1}{2}\right)\mathbf{a}_2 - x_9\,\mathbf{a}_3 \quad = \quad a\left(y_9 + \tfrac{1}{2}\right)\hat{\mathbf{x}} - a\left(z_9 - \tfrac{1}{2}\right)\hat{\mathbf{y}} - ax_9\,\hat{\mathbf{z}} \qquad (24\mathrm{d}) \qquad \mathrm{H\ IV}$$

$$\mathbf{B_{116}} = \quad -\left(y_9 - \tfrac{1}{2}\right)\mathbf{a}_1 - z_9\,\mathbf{a}_2 + \left(x_9 + \tfrac{1}{2}\right)\mathbf{a}_3 \quad = \quad -a\left(y_9 - \tfrac{1}{2}\right)\hat{\mathbf{x}} - az_9\,\hat{\mathbf{y}} + a\left(x_9 + \tfrac{1}{2}\right)\hat{\mathbf{z}} \qquad (24\mathrm{d}) \qquad \mathrm{H\ IV}$$

$$\mathbf{B_{117}} = \quad -x_9\,\mathbf{a}_1 - y_9\,\mathbf{a}_2 - z_9\,\mathbf{a}_3 \quad = \quad -ax_9\,\hat{\mathbf{x}} - ay_9\,\hat{\mathbf{y}} - az_9\,\hat{\mathbf{z}} \qquad (24\mathrm{d}) \qquad \mathrm{H\ IV}$$

$$\mathbf{B_{118}} = \quad \left(x_9 + \tfrac{1}{2}\right)\mathbf{a}_1 + y_9\,\mathbf{a}_2 - \left(z_9 - \tfrac{1}{2}\right)\mathbf{a}_3 \quad = \quad a\left(x_9 + \tfrac{1}{2}\right)\hat{\mathbf{x}} + ay_9\,\hat{\mathbf{y}} - a\left(z_9 - \tfrac{1}{2}\right)\hat{\mathbf{z}} \qquad (24\mathrm{d}) \qquad \mathrm{H\ IV}$$

$$\mathbf{B_{119}} = \quad x_9\,\mathbf{a}_1 - \left(y_9 - \tfrac{1}{2}\right)\mathbf{a}_2 + \left(z_9 + \tfrac{1}{2}\right)\mathbf{a}_3 \quad = \quad ax_9\,\hat{\mathbf{x}} - a\left(y_9 - \tfrac{1}{2}\right)\hat{\mathbf{y}} + a\left(z_9 + \tfrac{1}{2}\right)\hat{\mathbf{z}} \qquad (24\mathrm{d}) \qquad \mathrm{H\ IV}$$

$$\mathbf{B_{120}} = \quad -\left(x_9 - \tfrac{1}{2}\right)\mathbf{a}_1 + \left(y_9 + \tfrac{1}{2}\right)\mathbf{a}_2 + z_9\,\mathbf{a}_3 \quad = \quad -a\left(x_9 - \tfrac{1}{2}\right)\hat{\mathbf{x}} + a\left(y_9 + \tfrac{1}{2}\right)\hat{\mathbf{y}} + az_9\,\hat{\mathbf{z}} \qquad (24\mathrm{d}) \qquad \mathrm{H\ IV}$$

$$\mathbf{B_{121}} = \quad -z_9\,\mathbf{a}_1 - x_9\,\mathbf{a}_2 - y_9\,\mathbf{a}_3 \quad = \quad -az_9\,\hat{\mathbf{x}} - ax_9\,\hat{\mathbf{y}} - ay_9\,\hat{\mathbf{z}} \qquad (24\mathrm{d}) \qquad \mathrm{H\ IV}$$

$$\mathbf{B_{122}} = \quad -\left(z_9 - \tfrac{1}{2}\right)\mathbf{a}_1 + \left(x_9 + \tfrac{1}{2}\right)\mathbf{a}_2 + y_9\,\mathbf{a}_3 \quad = \quad -a\left(z_9 - \tfrac{1}{2}\right)\hat{\mathbf{x}} + a\left(x_9 + \tfrac{1}{2}\right)\hat{\mathbf{y}} + ay_9\,\hat{\mathbf{z}} \qquad (24\mathrm{d}) \qquad \mathrm{H\ IV}$$

$$\mathbf{B_{123}} = \quad \left(z_9 + \tfrac{1}{2}\right)\mathbf{a}_1 + x_9\,\mathbf{a}_2 - \left(y_9 - \tfrac{1}{2}\right)\mathbf{a}_3 \quad = \quad a\left(z_9 + \tfrac{1}{2}\right)\hat{\mathbf{x}} + ax_9\,\hat{\mathbf{y}} - a\left(y_9 - \tfrac{1}{2}\right)\hat{\mathbf{z}} \qquad (24\mathrm{d}) \qquad \mathrm{H\ IV}$$

$$\mathbf{B_{124}} = \quad z_9\,\mathbf{a}_1 - \left(x_9 - \tfrac{1}{2}\right)\mathbf{a}_2 + \left(y_9 + \tfrac{1}{2}\right)\mathbf{a}_3 \quad = \quad az_9\,\hat{\mathbf{x}} - a\left(x_9 - \tfrac{1}{2}\right)\hat{\mathbf{y}} + a\left(y_9 + \tfrac{1}{2}\right)\hat{\mathbf{z}} \qquad (24\mathrm{d}) \qquad \mathrm{H\ IV}$$

$$\mathbf{B_{125}} = \quad -y_9\,\mathbf{a}_1 - z_9\,\mathbf{a}_2 - x_9\,\mathbf{a}_3 \quad = \quad -ay_9\,\hat{\mathbf{x}} - az_9\,\hat{\mathbf{y}} - ax_9\,\hat{\mathbf{z}} \qquad (24\mathrm{d}) \qquad \mathrm{H\ IV}$$

$$\mathbf{B_{126}} = \quad y_9\,\mathbf{a}_1 - \left(z_9 - \tfrac{1}{2}\right)\mathbf{a}_2 + \left(x_9 + \tfrac{1}{2}\right)\mathbf{a}_3 \quad = \quad ay_9\,\hat{\mathbf{x}} - a\left(z_9 - \tfrac{1}{2}\right)\hat{\mathbf{y}} + a\left(x_9 + \tfrac{1}{2}\right)\hat{\mathbf{z}} \qquad (24\mathrm{d}) \qquad \mathrm{H\ IV}$$

$$\mathbf{B_{127}} = \quad -\left(y_9 - \tfrac{1}{2}\right)\mathbf{a}_1 + \left(z_9 + \tfrac{1}{2}\right)\mathbf{a}_2 + x_9\,\mathbf{a}_3 \quad = \quad -a\left(y_9 - \tfrac{1}{2}\right)\hat{\mathbf{x}} + a\left(z_9 + \tfrac{1}{2}\right)\hat{\mathbf{y}} + ax_9\,\hat{\mathbf{z}} \qquad (24\mathrm{d}) \qquad \mathrm{H\ IV}$$

$$\mathbf{B_{128}} = \quad \left(y_9 + \tfrac{1}{2}\right)\mathbf{a}_1 + z_9\,\mathbf{a}_2 - \left(x_9 - \tfrac{1}{2}\right)\mathbf{a}_3 \quad = \quad a\left(y_9 + \tfrac{1}{2}\right)\hat{\mathbf{x}} + az_9\,\hat{\mathbf{y}} - a\left(x_9 - \tfrac{1}{2}\right)\hat{\mathbf{z}} \qquad (24\mathrm{d}) \qquad \mathrm{H\ IV}$$

$$\mathbf{B_{129}} = \quad x_{10}\,\mathbf{a}_1 + y_{10}\,\mathbf{a}_2 + z_{10}\,\mathbf{a}_3 \quad = \quad ax_{10}\,\hat{\mathbf{x}} + ay_{10}\,\hat{\mathbf{y}} + az_{10}\,\hat{\mathbf{z}} \qquad (24\mathrm{d}) \qquad \mathrm{O\ III}$$

$$\mathbf{B_{130}} = \quad -\left(x_{10} - \tfrac{1}{2}\right)\mathbf{a}_1 - y_{10}\,\mathbf{a}_2 + \left(z_{10} + \tfrac{1}{2}\right)\mathbf{a}_3 \quad = \quad -a\left(x_{10} - \tfrac{1}{2}\right)\hat{\mathbf{x}} - ay_{10}\,\hat{\mathbf{y}} + a\left(z_{10} + \tfrac{1}{2}\right)\hat{\mathbf{z}} \qquad (24\mathrm{d}) \qquad \mathrm{O\ III}$$

$$\mathbf{B_{131}} = \quad -x_{10}\,\mathbf{a}_1 + \left(y_{10} + \tfrac{1}{2}\right)\mathbf{a}_2 - \left(z_{10} - \tfrac{1}{2}\right)\mathbf{a}_3 \quad = \quad -ax_{10}\,\hat{\mathbf{x}} + a\left(y_{10} + \tfrac{1}{2}\right)\hat{\mathbf{y}} - a\left(z_{10} - \tfrac{1}{2}\right)\hat{\mathbf{z}} \qquad (24\mathrm{d}) \qquad \mathrm{O\ III}$$

$$\mathbf{B_{132}} = \quad \left(x_{10} + \tfrac{1}{2}\right)\mathbf{a}_1 - \left(y_{10} - \tfrac{1}{2}\right)\mathbf{a}_2 - z_{10}\,\mathbf{a}_3 \quad = \quad a\left(x_{10} + \tfrac{1}{2}\right)\hat{\mathbf{x}} - a\left(y_{10} - \tfrac{1}{2}\right)\hat{\mathbf{y}} - az_{10}\,\hat{\mathbf{z}} \qquad (24\mathrm{d}) \qquad \mathrm{O\ III}$$

$$\mathbf{B_{133}} = \quad z_{10}\,\mathbf{a}_1 + x_{10}\,\mathbf{a}_2 + y_{10}\,\mathbf{a}_3 \quad = \quad az_{10}\,\hat{\mathbf{x}} + ax_{10}\,\hat{\mathbf{y}} + ay_{10}\,\hat{\mathbf{z}} \qquad (24\mathrm{d}) \qquad \mathrm{O\ III}$$

$$\mathbf{B_{134}} = \quad \left(z_{10} + \tfrac{1}{2}\right)\mathbf{a}_1 - \left(x_{10} - \tfrac{1}{2}\right)\mathbf{a}_2 - y_{10}\,\mathbf{a}_3 \quad = \quad a\left(z_{10} + \tfrac{1}{2}\right)\hat{\mathbf{x}} - a\left(x_{10} - \tfrac{1}{2}\right)\hat{\mathbf{y}} - ay_{10}\,\hat{\mathbf{z}} \qquad \text{(24d)} \qquad \text{O III}$$

$$\mathbf{B_{135}} = \quad -\left(z_{10} - \tfrac{1}{2}\right)\mathbf{a}_1 - x_{10}\,\mathbf{a}_2 + \left(y_{10} + \tfrac{1}{2}\right)\mathbf{a}_3 \quad = \quad -a\left(z_{10} - \tfrac{1}{2}\right)\hat{\mathbf{x}} - ax_{10}\,\hat{\mathbf{y}} + a\left(y_{10} + \tfrac{1}{2}\right)\hat{\mathbf{z}} \qquad \text{(24d)} \qquad \text{O III}$$

$$\mathbf{B_{136}} = \quad -z_{10}\,\mathbf{a}_1 + \left(x_{10} + \tfrac{1}{2}\right)\mathbf{a}_2 - \left(y_{10} - \tfrac{1}{2}\right)\mathbf{a}_3 \quad = \quad -az_{10}\,\hat{\mathbf{x}} + a\left(x_{10} + \tfrac{1}{2}\right)\hat{\mathbf{y}} - a\left(y_{10} - \tfrac{1}{2}\right)\hat{\mathbf{z}} \qquad \text{(24d)} \qquad \text{O III}$$

$$\mathbf{B_{137}} = \quad y_{10}\,\mathbf{a}_1 + z_{10}\,\mathbf{a}_2 + x_{10}\,\mathbf{a}_3 \quad = \quad ay_{10}\,\hat{\mathbf{x}} + az_{10}\,\hat{\mathbf{y}} + ax_{10}\,\hat{\mathbf{z}} \qquad \text{(24d)} \qquad \text{O III}$$

$$\mathbf{B_{138}} = \quad -y_{10}\,\mathbf{a}_1 + \left(z_{10} + \tfrac{1}{2}\right)\mathbf{a}_2 - \left(x_{10} - \tfrac{1}{2}\right)\mathbf{a}_3 \quad = \quad -ay_{10}\,\hat{\mathbf{x}} + a\left(z_{10} + \tfrac{1}{2}\right)\hat{\mathbf{y}} - a\left(x_{10} - \tfrac{1}{2}\right)\hat{\mathbf{z}} \qquad \text{(24d)} \qquad \text{O III}$$

$$\mathbf{B_{139}} = \quad \left(y_{10} + \tfrac{1}{2}\right)\mathbf{a}_1 - \left(z_{10} - \tfrac{1}{2}\right)\mathbf{a}_2 - x_{10}\,\mathbf{a}_3 \quad = \quad a\left(y_{10} + \tfrac{1}{2}\right)\hat{\mathbf{x}} - a\left(z_{10} - \tfrac{1}{2}\right)\hat{\mathbf{y}} - ax_{10}\,\hat{\mathbf{z}} \qquad \text{(24d)} \qquad \text{O III}$$

$$\mathbf{B_{140}} = \quad -\left(y_{10} - \tfrac{1}{2}\right)\mathbf{a}_1 - z_{10}\,\mathbf{a}_2 + \left(x_{10} + \tfrac{1}{2}\right)\mathbf{a}_3 \quad = \quad -a\left(y_{10} - \tfrac{1}{2}\right)\hat{\mathbf{x}} - az_{10}\,\hat{\mathbf{y}} + a\left(x_{10} + \tfrac{1}{2}\right)\hat{\mathbf{z}} \qquad \text{(24d)} \qquad \text{O III}$$

$$\mathbf{B_{141}} = \quad -x_{10}\,\mathbf{a}_1 - y_{10}\,\mathbf{a}_2 - z_{10}\,\mathbf{a}_3 \quad = \quad -ax_{10}\,\hat{\mathbf{x}} - ay_{10}\,\hat{\mathbf{y}} - az_{10}\,\hat{\mathbf{z}} \qquad \text{(24d)} \qquad \text{O III}$$

$$\mathbf{B_{142}} = \quad \left(x_{10} + \tfrac{1}{2}\right)\mathbf{a}_1 + y_{10}\,\mathbf{a}_2 - \left(z_{10} - \tfrac{1}{2}\right)\mathbf{a}_3 \quad = \quad a\left(x_{10} + \tfrac{1}{2}\right)\hat{\mathbf{x}} + ay_{10}\,\hat{\mathbf{y}} - a\left(z_{10} - \tfrac{1}{2}\right)\hat{\mathbf{z}} \qquad \text{(24d)} \qquad \text{O III}$$

$$\mathbf{B_{143}} = \quad x_{10}\,\mathbf{a}_1 - \left(y_{10} - \tfrac{1}{2}\right)\mathbf{a}_2 + \left(z_{10} + \tfrac{1}{2}\right)\mathbf{a}_3 \quad = \quad ax_{10}\,\hat{\mathbf{x}} - a\left(y_{10} - \tfrac{1}{2}\right)\hat{\mathbf{y}} + a\left(z_{10} + \tfrac{1}{2}\right)\hat{\mathbf{z}} \qquad \text{(24d)} \qquad \text{O III}$$

$$\mathbf{B_{144}} = \quad -\left(x_{10} - \tfrac{1}{2}\right)\mathbf{a}_1 + \left(y_{10} + \tfrac{1}{2}\right)\mathbf{a}_2 + z_{10}\,\mathbf{a}_3 \quad = \quad -a\left(x_{10} - \tfrac{1}{2}\right)\hat{\mathbf{x}} + a\left(y_{10} + \tfrac{1}{2}\right)\hat{\mathbf{y}} + az_{10}\,\hat{\mathbf{z}} \qquad \text{(24d)} \qquad \text{O III}$$

$$\mathbf{B_{145}} = \quad -z_{10}\,\mathbf{a}_1 - x_{10}\,\mathbf{a}_2 - y_{10}\,\mathbf{a}_3 \quad = \quad -az_{10}\,\hat{\mathbf{x}} - ax_{10}\,\hat{\mathbf{y}} - ay_{10}\,\hat{\mathbf{z}} \qquad \text{(24d)} \qquad \text{O III}$$

$$\mathbf{B_{146}} = \quad -\left(z_{10} - \tfrac{1}{2}\right)\mathbf{a}_1 + \left(x_{10} + \tfrac{1}{2}\right)\mathbf{a}_2 + y_{10}\,\mathbf{a}_3 \quad = \quad -a\left(z_{10} - \tfrac{1}{2}\right)\hat{\mathbf{x}} + a\left(x_{10} + \tfrac{1}{2}\right)\hat{\mathbf{y}} + ay_{10}\,\hat{\mathbf{z}} \qquad \text{(24d)} \qquad \text{O III}$$

$$\mathbf{B_{147}} = \quad \left(z_{10} + \tfrac{1}{2}\right)\mathbf{a}_1 + x_{10}\,\mathbf{a}_2 - \left(y_{10} - \tfrac{1}{2}\right)\mathbf{a}_3 \quad = \quad a\left(z_{10} + \tfrac{1}{2}\right)\hat{\mathbf{x}} + ax_{10}\,\hat{\mathbf{y}} - a\left(y_{10} - \tfrac{1}{2}\right)\hat{\mathbf{z}} \qquad \text{(24d)} \qquad \text{O III}$$

$$\mathbf{B_{148}} = \quad z_{10}\,\mathbf{a}_1 - \left(x_{10} - \tfrac{1}{2}\right)\mathbf{a}_2 + \left(y_{10} + \tfrac{1}{2}\right)\mathbf{a}_3 \quad = \quad az_{10}\,\hat{\mathbf{x}} - a\left(x_{10} - \tfrac{1}{2}\right)\hat{\mathbf{y}} + a\left(y_{10} + \tfrac{1}{2}\right)\hat{\mathbf{z}} \qquad \text{(24d)} \qquad \text{O III}$$

$$\mathbf{B_{149}} = \quad -y_{10}\,\mathbf{a}_1 - z_{10}\,\mathbf{a}_2 - x_{10}\,\mathbf{a}_3 \quad = \quad -ay_{10}\,\hat{\mathbf{x}} - az_{10}\,\hat{\mathbf{y}} - ax_{10}\,\hat{\mathbf{z}} \qquad \text{(24d)} \qquad \text{O III}$$

$$\mathbf{B_{150}} = \quad y_{10}\,\mathbf{a}_1 - \left(z_{10} - \tfrac{1}{2}\right)\mathbf{a}_2 + \left(x_{10} + \tfrac{1}{2}\right)\mathbf{a}_3 \quad = \quad ay_{10}\,\hat{\mathbf{x}} - a\left(z_{10} - \tfrac{1}{2}\right)\hat{\mathbf{y}} + a\left(x_{10} + \tfrac{1}{2}\right)\hat{\mathbf{z}} \qquad \text{(24d)} \qquad \text{O III}$$

$$\mathbf{B_{151}} = \quad -\left(y_{10} - \tfrac{1}{2}\right)\mathbf{a}_1 + \left(z_{10} + \tfrac{1}{2}\right)\mathbf{a}_2 + x_{10}\,\mathbf{a}_3 \quad = \quad -a\left(y_{10} - \tfrac{1}{2}\right)\hat{\mathbf{x}} + a\left(z_{10} + \tfrac{1}{2}\right)\hat{\mathbf{y}} + ax_{10}\,\hat{\mathbf{z}} \qquad \text{(24d)} \qquad \text{O III}$$

$$\mathbf{B_{152}} = \quad \left(y_{10} + \tfrac{1}{2}\right)\mathbf{a}_1 + z_{10}\,\mathbf{a}_2 - \left(x_{10} - \tfrac{1}{2}\right)\mathbf{a}_3 \quad = \quad a\left(y_{10} + \tfrac{1}{2}\right)\hat{\mathbf{x}} + az_{10}\,\hat{\mathbf{y}} - a\left(x_{10} - \tfrac{1}{2}\right)\hat{\mathbf{z}} \qquad \text{(24d)} \qquad \text{O III}$$

$$\mathbf{B_{153}} = \quad x_{11}\,\mathbf{a}_1 + y_{11}\,\mathbf{a}_2 + z_{11}\,\mathbf{a}_3 \quad = \quad ax_{11}\,\hat{\mathbf{x}} + ay_{11}\,\hat{\mathbf{y}} + az_{11}\,\hat{\mathbf{z}} \qquad \text{(24d)} \qquad \text{O IV}$$

$$\mathbf{B_{154}} = \quad -\left(x_{11} - \tfrac{1}{2}\right)\mathbf{a}_1 - y_{11}\,\mathbf{a}_2 + \left(z_{11} + \tfrac{1}{2}\right)\mathbf{a}_3 \quad = \quad -a\left(x_{11} - \tfrac{1}{2}\right)\hat{\mathbf{x}} - ay_{11}\,\hat{\mathbf{y}} + a\left(z_{11} + \tfrac{1}{2}\right)\hat{\mathbf{z}} \qquad \text{(24d)} \qquad \text{O IV}$$

$$\mathbf{B_{155}} = \quad -x_{11}\,\mathbf{a}_1 + \left(y_{11} + \tfrac{1}{2}\right)\mathbf{a}_2 - \left(z_{11} - \tfrac{1}{2}\right)\mathbf{a}_3 \quad = \quad -ax_{11}\,\hat{\mathbf{x}} + a\left(y_{11} + \tfrac{1}{2}\right)\hat{\mathbf{y}} - a\left(z_{11} - \tfrac{1}{2}\right)\hat{\mathbf{z}} \qquad \text{(24d)} \qquad \text{O IV}$$

$$\mathbf{B_{156}} = \quad \left(x_{11} + \tfrac{1}{2}\right)\mathbf{a}_1 - \left(y_{11} - \tfrac{1}{2}\right)\mathbf{a}_2 - z_{11}\,\mathbf{a}_3 \quad = \quad a\left(x_{11} + \tfrac{1}{2}\right)\hat{\mathbf{x}} - a\left(y_{11} - \tfrac{1}{2}\right)\hat{\mathbf{y}} - az_{11}\,\hat{\mathbf{z}} \qquad \text{(24d)} \qquad \text{O IV}$$

$$\mathbf{B_{157}} = \quad z_{11}\,\mathbf{a}_1 + x_{11}\,\mathbf{a}_2 + y_{11}\,\mathbf{a}_3 \quad = \quad az_{11}\,\hat{\mathbf{x}} + ax_{11}\,\hat{\mathbf{y}} + ay_{11}\,\hat{\mathbf{z}} \qquad \text{(24d)} \qquad \text{O IV}$$

$$\mathbf{B_{158}} = \quad \left(z_{11} + \tfrac{1}{2}\right)\mathbf{a}_1 - \left(x_{11} - \tfrac{1}{2}\right)\mathbf{a}_2 - y_{11}\,\mathbf{a}_3 \quad = \quad a\left(z_{11} + \tfrac{1}{2}\right)\hat{\mathbf{x}} - a\left(x_{11} - \tfrac{1}{2}\right)\hat{\mathbf{y}} - ay_{11}\,\hat{\mathbf{z}} \qquad \text{(24d)} \qquad \text{O IV}$$

$$\mathbf{B_{159}} = \quad -\left(z_{11} - \tfrac{1}{2}\right)\mathbf{a}_1 - x_{11}\,\mathbf{a}_2 + \left(y_{11} + \tfrac{1}{2}\right)\mathbf{a}_3 \quad = \quad -a\left(z_{11} - \tfrac{1}{2}\right)\hat{\mathbf{x}} - ax_{11}\,\hat{\mathbf{y}} + a\left(y_{11} + \tfrac{1}{2}\right)\hat{\mathbf{z}} \qquad \text{(24d)} \qquad \text{O IV}$$

$$\mathbf{B_{160}} = -z_{11}\,\mathbf{a}_1 + \left(x_{11} + \tfrac{1}{2}\right)\mathbf{a}_2 - \left(y_{11} - \tfrac{1}{2}\right)\mathbf{a}_3 = -az_{11}\,\hat{\mathbf{x}} + a\left(x_{11} + \tfrac{1}{2}\right)\hat{\mathbf{y}} - a\left(y_{11} - \tfrac{1}{2}\right)\hat{\mathbf{z}} \quad (24\mathrm{d}) \quad \text{O IV}$$

$$\mathbf{B_{161}} = y_{11}\,\mathbf{a}_1 + z_{11}\,\mathbf{a}_2 + x_{11}\,\mathbf{a}_3 = ay_{11}\,\hat{\mathbf{x}} + az_{11}\,\hat{\mathbf{y}} + ax_{11}\,\hat{\mathbf{z}} \quad (24\mathrm{d}) \quad \text{O IV}$$

$$\mathbf{B_{162}} = -y_{11}\,\mathbf{a}_1 + \left(z_{11} + \tfrac{1}{2}\right)\mathbf{a}_2 - \left(x_{11} - \tfrac{1}{2}\right)\mathbf{a}_3 = -ay_{11}\,\hat{\mathbf{x}} + a\left(z_{11} + \tfrac{1}{2}\right)\hat{\mathbf{y}} - a\left(x_{11} - \tfrac{1}{2}\right)\hat{\mathbf{z}} \quad (24\mathrm{d}) \quad \text{O IV}$$

$$\mathbf{B_{163}} = \left(y_{11} + \tfrac{1}{2}\right)\mathbf{a}_1 - \left(z_{11} - \tfrac{1}{2}\right)\mathbf{a}_2 - x_{11}\,\mathbf{a}_3 = a\left(y_{11} + \tfrac{1}{2}\right)\hat{\mathbf{x}} - a\left(z_{11} - \tfrac{1}{2}\right)\hat{\mathbf{y}} - ax_{11}\,\hat{\mathbf{z}} \quad (24\mathrm{d}) \quad \text{O IV}$$

$$\mathbf{B_{164}} = -\left(y_{11} - \tfrac{1}{2}\right)\mathbf{a}_1 - z_{11}\,\mathbf{a}_2 + \left(x_{11} + \tfrac{1}{2}\right)\mathbf{a}_3 = -a\left(y_{11} - \tfrac{1}{2}\right)\hat{\mathbf{x}} - az_{11}\,\hat{\mathbf{y}} + a\left(x_{11} + \tfrac{1}{2}\right)\hat{\mathbf{z}} \quad (24\mathrm{d}) \quad \text{O IV}$$

$$\mathbf{B_{165}} = -x_{11}\,\mathbf{a}_1 - y_{11}\,\mathbf{a}_2 - z_{11}\,\mathbf{a}_3 = -ax_{11}\,\hat{\mathbf{x}} - ay_{11}\,\hat{\mathbf{y}} - az_{11}\,\hat{\mathbf{z}} \quad (24\mathrm{d}) \quad \text{O IV}$$

$$\mathbf{B_{166}} = \left(x_{11} + \tfrac{1}{2}\right)\mathbf{a}_1 + y_{11}\,\mathbf{a}_2 - \left(z_{11} - \tfrac{1}{2}\right)\mathbf{a}_3 = a\left(x_{11} + \tfrac{1}{2}\right)\hat{\mathbf{x}} + ay_{11}\,\hat{\mathbf{y}} - a\left(z_{11} - \tfrac{1}{2}\right)\hat{\mathbf{z}} \quad (24\mathrm{d}) \quad \text{O IV}$$

$$\mathbf{B_{167}} = x_{11}\,\mathbf{a}_1 - \left(y_{11} - \tfrac{1}{2}\right)\mathbf{a}_2 + \left(z_{11} + \tfrac{1}{2}\right)\mathbf{a}_3 = ax_{11}\,\hat{\mathbf{x}} - a\left(y_{11} - \tfrac{1}{2}\right)\hat{\mathbf{y}} + a\left(z_{11} + \tfrac{1}{2}\right)\hat{\mathbf{z}} \quad (24\mathrm{d}) \quad \text{O IV}$$

$$\mathbf{B_{168}} = -\left(x_{11} - \tfrac{1}{2}\right)\mathbf{a}_1 + \left(y_{11} + \tfrac{1}{2}\right)\mathbf{a}_2 + z_{11}\,\mathbf{a}_3 = -a\left(x_{11} - \tfrac{1}{2}\right)\hat{\mathbf{x}} + a\left(y_{11} + \tfrac{1}{2}\right)\hat{\mathbf{y}} + az_{11}\,\hat{\mathbf{z}} \quad (24\mathrm{d}) \quad \text{O IV}$$

$$\mathbf{B_{169}} = -z_{11}\,\mathbf{a}_1 - x_{11}\,\mathbf{a}_2 - y_{11}\,\mathbf{a}_3 = -az_{11}\,\hat{\mathbf{x}} - ax_{11}\,\hat{\mathbf{y}} - ay_{11}\,\hat{\mathbf{z}} \quad (24\mathrm{d}) \quad \text{O IV}$$

$$\mathbf{B_{170}} = -\left(z_{11} - \tfrac{1}{2}\right)\mathbf{a}_1 + \left(x_{11} + \tfrac{1}{2}\right)\mathbf{a}_2 + y_{11}\,\mathbf{a}_3 = -a\left(z_{11} - \tfrac{1}{2}\right)\hat{\mathbf{x}} + a\left(x_{11} + \tfrac{1}{2}\right)\hat{\mathbf{y}} + ay_{11}\,\hat{\mathbf{z}} \quad (24\mathrm{d}) \quad \text{O IV}$$

$$\mathbf{B_{171}} = \left(z_{11} + \tfrac{1}{2}\right)\mathbf{a}_1 + x_{11}\,\mathbf{a}_2 - \left(y_{11} - \tfrac{1}{2}\right)\mathbf{a}_3 = a\left(z_{11} + \tfrac{1}{2}\right)\hat{\mathbf{x}} + ax_{11}\,\hat{\mathbf{y}} - a\left(y_{11} - \tfrac{1}{2}\right)\hat{\mathbf{z}} \quad (24\mathrm{d}) \quad \text{O IV}$$

$$\mathbf{B_{172}} = z_{11}\,\mathbf{a}_1 - \left(x_{11} - \tfrac{1}{2}\right)\mathbf{a}_2 + \left(y_{11} + \tfrac{1}{2}\right)\mathbf{a}_3 = az_{11}\,\hat{\mathbf{x}} - a\left(x_{11} - \tfrac{1}{2}\right)\hat{\mathbf{y}} + a\left(y_{11} + \tfrac{1}{2}\right)\hat{\mathbf{z}} \quad (24\mathrm{d}) \quad \text{O IV}$$

$$\mathbf{B_{173}} = -y_{11}\,\mathbf{a}_1 - z_{11}\,\mathbf{a}_2 - x_{11}\,\mathbf{a}_3 = -ay_{11}\,\hat{\mathbf{x}} - az_{11}\,\hat{\mathbf{y}} - ax_{11}\,\hat{\mathbf{z}} \quad (24\mathrm{d}) \quad \text{O IV}$$

$$\mathbf{B_{174}} = y_{11}\,\mathbf{a}_1 - \left(z_{11} - \tfrac{1}{2}\right)\mathbf{a}_2 + \left(x_{11} + \tfrac{1}{2}\right)\mathbf{a}_3 = ay_{11}\,\hat{\mathbf{x}} - a\left(z_{11} - \tfrac{1}{2}\right)\hat{\mathbf{y}} + a\left(x_{11} + \tfrac{1}{2}\right)\hat{\mathbf{z}} \quad (24\mathrm{d}) \quad \text{O IV}$$

$$\mathbf{B_{175}} = -\left(y_{11} - \tfrac{1}{2}\right)\mathbf{a}_1 + \left(z_{11} + \tfrac{1}{2}\right)\mathbf{a}_2 + x_{11}\,\mathbf{a}_3 = -a\left(y_{11} - \tfrac{1}{2}\right)\hat{\mathbf{x}} + a\left(z_{11} + \tfrac{1}{2}\right)\hat{\mathbf{y}} + ax_{11}\,\hat{\mathbf{z}} \quad (24\mathrm{d}) \quad \text{O IV}$$

$$\mathbf{B_{176}} = \left(y_{11} + \tfrac{1}{2}\right)\mathbf{a}_1 + z_{11}\,\mathbf{a}_2 - \left(x_{11} - \tfrac{1}{2}\right)\mathbf{a}_3 = a\left(y_{11} + \tfrac{1}{2}\right)\hat{\mathbf{x}} + az_{11}\,\hat{\mathbf{y}} - a\left(x_{11} - \tfrac{1}{2}\right)\hat{\mathbf{z}} \quad (24\mathrm{d}) \quad \text{O IV}$$

$$\mathbf{B_{177}} = x_{12}\,\mathbf{a}_1 + y_{12}\,\mathbf{a}_2 + z_{12}\,\mathbf{a}_3 = ax_{12}\,\hat{\mathbf{x}} + ay_{12}\,\hat{\mathbf{y}} + az_{12}\,\hat{\mathbf{z}} \quad (24\mathrm{d}) \quad \text{O V}$$

$$\mathbf{B_{178}} = -\left(x_{12} - \tfrac{1}{2}\right)\mathbf{a}_1 - y_{12}\,\mathbf{a}_2 + \left(z_{12} + \tfrac{1}{2}\right)\mathbf{a}_3 = -a\left(x_{12} - \tfrac{1}{2}\right)\hat{\mathbf{x}} - ay_{12}\,\hat{\mathbf{y}} + a\left(z_{12} + \tfrac{1}{2}\right)\hat{\mathbf{z}} \quad (24\mathrm{d}) \quad \text{O V}$$

$$\mathbf{B_{179}} = -x_{12}\,\mathbf{a}_1 + \left(y_{12} + \tfrac{1}{2}\right)\mathbf{a}_2 - \left(z_{12} - \tfrac{1}{2}\right)\mathbf{a}_3 = -ax_{12}\,\hat{\mathbf{x}} + a\left(y_{12} + \tfrac{1}{2}\right)\hat{\mathbf{y}} - a\left(z_{12} - \tfrac{1}{2}\right)\hat{\mathbf{z}} \quad (24\mathrm{d}) \quad \text{O V}$$

$$\mathbf{B_{180}} = \left(x_{12} + \tfrac{1}{2}\right)\mathbf{a}_1 - \left(y_{12} - \tfrac{1}{2}\right)\mathbf{a}_2 - z_{12}\,\mathbf{a}_3 = a\left(x_{12} + \tfrac{1}{2}\right)\hat{\mathbf{x}} - a\left(y_{12} - \tfrac{1}{2}\right)\hat{\mathbf{y}} - az_{12}\,\hat{\mathbf{z}} \quad (24\mathrm{d}) \quad \text{O V}$$

$$\mathbf{B_{181}} = z_{12}\,\mathbf{a}_1 + x_{12}\,\mathbf{a}_2 + y_{12}\,\mathbf{a}_3 = az_{12}\,\hat{\mathbf{x}} + ax_{12}\,\hat{\mathbf{y}} + ay_{12}\,\hat{\mathbf{z}} \quad (24\mathrm{d}) \quad \text{O V}$$

$$\mathbf{B_{182}} = \left(z_{12} + \tfrac{1}{2}\right)\mathbf{a}_1 - \left(x_{12} - \tfrac{1}{2}\right)\mathbf{a}_2 - y_{12}\,\mathbf{a}_3 = a\left(z_{12} + \tfrac{1}{2}\right)\hat{\mathbf{x}} - a\left(x_{12} - \tfrac{1}{2}\right)\hat{\mathbf{y}} - ay_{12}\,\hat{\mathbf{z}} \quad (24\mathrm{d}) \quad \text{O V}$$

$$\mathbf{B_{183}} = -\left(z_{12} - \tfrac{1}{2}\right)\mathbf{a}_1 - x_{12}\,\mathbf{a}_2 + \left(y_{12} + \tfrac{1}{2}\right)\mathbf{a}_3 = -a\left(z_{12} - \tfrac{1}{2}\right)\hat{\mathbf{x}} - ax_{12}\,\hat{\mathbf{y}} + a\left(y_{12} + \tfrac{1}{2}\right)\hat{\mathbf{z}} \quad (24\mathrm{d}) \quad \text{O V}$$

$$\mathbf{B_{184}} = -z_{12}\,\mathbf{a}_1 + \left(x_{12} + \tfrac{1}{2}\right)\mathbf{a}_2 - \left(y_{12} - \tfrac{1}{2}\right)\mathbf{a}_3 = -az_{12}\,\hat{\mathbf{x}} + a\left(x_{12} + \tfrac{1}{2}\right)\hat{\mathbf{y}} - a\left(y_{12} - \tfrac{1}{2}\right)\hat{\mathbf{z}} \quad (24\mathrm{d}) \quad \text{O V}$$

$$\mathbf{B_{185}} = y_{12}\,\mathbf{a}_1 + z_{12}\,\mathbf{a}_2 + x_{12}\,\mathbf{a}_3 = ay_{12}\,\hat{\mathbf{x}} + az_{12}\,\hat{\mathbf{y}} + ax_{12}\,\hat{\mathbf{z}} \quad (24\mathrm{d}) \quad \text{O V}$$

$$\mathbf{B_{186}} = \quad -y_{12}\,\mathbf{a}_1 + \left(z_{12} + \tfrac{1}{2}\right)\mathbf{a}_2 - \left(x_{12} - \tfrac{1}{2}\right)\mathbf{a}_3 \quad = \quad -ay_{12}\,\hat{\mathbf{x}} + a\left(z_{12} + \tfrac{1}{2}\right)\hat{\mathbf{y}} - a\left(x_{12} - \tfrac{1}{2}\right)\hat{\mathbf{z}} \qquad \text{(24d)} \qquad \text{O V}$$

$$\mathbf{B_{187}} = \quad \left(y_{12} + \tfrac{1}{2}\right)\mathbf{a}_1 - \left(z_{12} - \tfrac{1}{2}\right)\mathbf{a}_2 - x_{12}\,\mathbf{a}_3 \quad = \quad a\left(y_{12} + \tfrac{1}{2}\right)\hat{\mathbf{x}} - a\left(z_{12} - \tfrac{1}{2}\right)\hat{\mathbf{y}} - ax_{12}\,\hat{\mathbf{z}} \qquad \text{(24d)} \qquad \text{O V}$$

$$\mathbf{B_{188}} = \quad -\left(y_{12} - \tfrac{1}{2}\right)\mathbf{a}_1 - z_{12}\,\mathbf{a}_2 + \left(x_{12} + \tfrac{1}{2}\right)\mathbf{a}_3 \quad = \quad -a\left(y_{12} - \tfrac{1}{2}\right)\hat{\mathbf{x}} - az_{12}\,\hat{\mathbf{y}} + a\left(x_{12} + \tfrac{1}{2}\right)\hat{\mathbf{z}} \qquad \text{(24d)} \qquad \text{O V}$$

$$\mathbf{B_{189}} = \quad -x_{12}\,\mathbf{a}_1 - y_{12}\,\mathbf{a}_2 - z_{12}\,\mathbf{a}_3 \quad = \quad -ax_{12}\,\hat{\mathbf{x}} - ay_{12}\,\hat{\mathbf{y}} - az_{12}\,\hat{\mathbf{z}} \qquad \text{(24d)} \qquad \text{O V}$$

$$\mathbf{B_{190}} = \quad \left(x_{12} + \tfrac{1}{2}\right)\mathbf{a}_1 + y_{12}\,\mathbf{a}_2 - \left(z_{12} - \tfrac{1}{2}\right)\mathbf{a}_3 \quad = \quad a\left(x_{12} + \tfrac{1}{2}\right)\hat{\mathbf{x}} + ay_{12}\,\hat{\mathbf{y}} - a\left(z_{12} - \tfrac{1}{2}\right)\hat{\mathbf{z}} \qquad \text{(24d)} \qquad \text{O V}$$

$$\mathbf{B_{191}} = \quad x_{12}\,\mathbf{a}_1 - \left(y_{12} - \tfrac{1}{2}\right)\mathbf{a}_2 + \left(z_{12} + \tfrac{1}{2}\right)\mathbf{a}_3 \quad = \quad ax_{12}\,\hat{\mathbf{x}} - a\left(y_{12} - \tfrac{1}{2}\right)\hat{\mathbf{y}} + a\left(z_{12} + \tfrac{1}{2}\right)\hat{\mathbf{z}} \qquad \text{(24d)} \qquad \text{O V}$$

$$\mathbf{B_{192}} = \quad -\left(x_{12} - \tfrac{1}{2}\right)\mathbf{a}_1 + \left(y_{12} + \tfrac{1}{2}\right)\mathbf{a}_2 + z_{12}\,\mathbf{a}_3 \quad = \quad -a\left(x_{12} - \tfrac{1}{2}\right)\hat{\mathbf{x}} + a\left(y_{12} + \tfrac{1}{2}\right)\hat{\mathbf{y}} + az_{12}\,\hat{\mathbf{z}} \qquad \text{(24d)} \qquad \text{O V}$$

$$\mathbf{B_{193}} = \quad -z_{12}\,\mathbf{a}_1 - x_{12}\,\mathbf{a}_2 - y_{12}\,\mathbf{a}_3 \quad = \quad -az_{12}\,\hat{\mathbf{x}} - ax_{12}\,\hat{\mathbf{y}} - ay_{12}\,\hat{\mathbf{z}} \qquad \text{(24d)} \qquad \text{O V}$$

$$\mathbf{B_{194}} = \quad -\left(z_{12} - \tfrac{1}{2}\right)\mathbf{a}_1 + \left(x_{12} + \tfrac{1}{2}\right)\mathbf{a}_2 + y_{12}\,\mathbf{a}_3 \quad = \quad -a\left(z_{12} - \tfrac{1}{2}\right)\hat{\mathbf{x}} + a\left(x_{12} + \tfrac{1}{2}\right)\hat{\mathbf{y}} + ay_{12}\,\hat{\mathbf{z}} \qquad \text{(24d)} \qquad \text{O V}$$

$$\mathbf{B_{195}} = \quad \left(z_{12} + \tfrac{1}{2}\right)\mathbf{a}_1 + x_{12}\,\mathbf{a}_2 - \left(y_{12} - \tfrac{1}{2}\right)\mathbf{a}_3 \quad = \quad a\left(z_{12} + \tfrac{1}{2}\right)\hat{\mathbf{x}} + ax_{12}\,\hat{\mathbf{y}} - a\left(y_{12} - \tfrac{1}{2}\right)\hat{\mathbf{z}} \qquad \text{(24d)} \qquad \text{O V}$$

$$\mathbf{B_{196}} = \quad z_{12}\,\mathbf{a}_1 - \left(x_{12} - \tfrac{1}{2}\right)\mathbf{a}_2 + \left(y_{12} + \tfrac{1}{2}\right)\mathbf{a}_3 \quad = \quad az_{12}\,\hat{\mathbf{x}} - a\left(x_{12} - \tfrac{1}{2}\right)\hat{\mathbf{y}} + a\left(y_{12} + \tfrac{1}{2}\right)\hat{\mathbf{z}} \qquad \text{(24d)} \qquad \text{O V}$$

$$\mathbf{B_{197}} = \quad -y_{12}\,\mathbf{a}_1 - z_{12}\,\mathbf{a}_2 - x_{12}\,\mathbf{a}_3 \quad = \quad -ay_{12}\,\hat{\mathbf{x}} - az_{12}\,\hat{\mathbf{y}} - ax_{12}\,\hat{\mathbf{z}} \qquad \text{(24d)} \qquad \text{O V}$$

$$\mathbf{B_{198}} = \quad y_{12}\,\mathbf{a}_1 - \left(z_{12} - \tfrac{1}{2}\right)\mathbf{a}_2 + \left(x_{12} + \tfrac{1}{2}\right)\mathbf{a}_3 \quad = \quad ay_{12}\,\hat{\mathbf{x}} - a\left(z_{12} - \tfrac{1}{2}\right)\hat{\mathbf{y}} + a\left(x_{12} + \tfrac{1}{2}\right)\hat{\mathbf{z}} \qquad \text{(24d)} \qquad \text{O V}$$

$$\mathbf{B_{199}} = \quad -\left(y_{12} - \tfrac{1}{2}\right)\mathbf{a}_1 + \left(z_{12} + \tfrac{1}{2}\right)\mathbf{a}_2 + x_{12}\,\mathbf{a}_3 \quad = \quad -a\left(y_{12} - \tfrac{1}{2}\right)\hat{\mathbf{x}} + a\left(z_{12} + \tfrac{1}{2}\right)\hat{\mathbf{y}} + ax_{12}\,\hat{\mathbf{z}} \qquad \text{(24d)} \qquad \text{O V}$$

$$\mathbf{B_{200}} = \quad \left(y_{12} + \tfrac{1}{2}\right)\mathbf{a}_1 + z_{12}\,\mathbf{a}_2 - \left(x_{12} - \tfrac{1}{2}\right)\mathbf{a}_3 \quad = \quad a\left(y_{12} + \tfrac{1}{2}\right)\hat{\mathbf{x}} + az_{12}\,\hat{\mathbf{y}} - a\left(x_{12} - \tfrac{1}{2}\right)\hat{\mathbf{z}} \qquad \text{(24d)} \qquad \text{O V}$$

$$\mathbf{B_{201}} = \quad x_{13}\,\mathbf{a}_1 + y_{13}\,\mathbf{a}_2 + z_{13}\,\mathbf{a}_3 \quad = \quad ax_{13}\,\hat{\mathbf{x}} + ay_{13}\,\hat{\mathbf{y}} + az_{13}\,\hat{\mathbf{z}} \qquad \text{(24d)} \qquad \text{O VI}$$

$$\mathbf{B_{202}} = \quad -\left(x_{13} - \tfrac{1}{2}\right)\mathbf{a}_1 - y_{13}\,\mathbf{a}_2 + \left(z_{13} + \tfrac{1}{2}\right)\mathbf{a}_3 \quad = \quad -a\left(x_{13} - \tfrac{1}{2}\right)\hat{\mathbf{x}} - ay_{13}\,\hat{\mathbf{y}} + a\left(z_{13} + \tfrac{1}{2}\right)\hat{\mathbf{z}} \qquad \text{(24d)} \qquad \text{O VI}$$

$$\mathbf{B_{203}} = \quad -x_{13}\,\mathbf{a}_1 + \left(y_{13} + \tfrac{1}{2}\right)\mathbf{a}_2 - \left(z_{13} - \tfrac{1}{2}\right)\mathbf{a}_3 \quad = \quad -ax_{13}\,\hat{\mathbf{x}} + a\left(y_{13} + \tfrac{1}{2}\right)\hat{\mathbf{y}} - a\left(z_{13} - \tfrac{1}{2}\right)\hat{\mathbf{z}} \qquad \text{(24d)} \qquad \text{O VI}$$

$$\mathbf{B_{204}} = \quad \left(x_{13} + \tfrac{1}{2}\right)\mathbf{a}_1 - \left(y_{13} - \tfrac{1}{2}\right)\mathbf{a}_2 - z_{13}\,\mathbf{a}_3 \quad = \quad a\left(x_{13} + \tfrac{1}{2}\right)\hat{\mathbf{x}} - a\left(y_{13} - \tfrac{1}{2}\right)\hat{\mathbf{y}} - az_{13}\,\hat{\mathbf{z}} \qquad \text{(24d)} \qquad \text{O VI}$$

$$\mathbf{B_{205}} = \quad z_{13}\,\mathbf{a}_1 + x_{13}\,\mathbf{a}_2 + y_{13}\,\mathbf{a}_3 \quad = \quad az_{13}\,\hat{\mathbf{x}} + ax_{13}\,\hat{\mathbf{y}} + ay_{13}\,\hat{\mathbf{z}} \qquad \text{(24d)} \qquad \text{O VI}$$

$$\mathbf{B_{206}} = \quad \left(z_{13} + \tfrac{1}{2}\right)\mathbf{a}_1 - \left(x_{13} - \tfrac{1}{2}\right)\mathbf{a}_2 - y_{13}\,\mathbf{a}_3 \quad = \quad a\left(z_{13} + \tfrac{1}{2}\right)\hat{\mathbf{x}} - a\left(x_{13} - \tfrac{1}{2}\right)\hat{\mathbf{y}} - ay_{13}\,\hat{\mathbf{z}} \qquad \text{(24d)} \qquad \text{O VI}$$

$$\mathbf{B_{207}} = \quad -\left(z_{13} - \tfrac{1}{2}\right)\mathbf{a}_1 - x_{13}\,\mathbf{a}_2 + \left(y_{13} + \tfrac{1}{2}\right)\mathbf{a}_3 \quad = \quad -a\left(z_{13} - \tfrac{1}{2}\right)\hat{\mathbf{x}} - ax_{13}\,\hat{\mathbf{y}} + a\left(y_{13} + \tfrac{1}{2}\right)\hat{\mathbf{z}} \qquad \text{(24d)} \qquad \text{O VI}$$

$$\mathbf{B_{208}} = \quad -z_{13}\,\mathbf{a}_1 + \left(x_{13} + \tfrac{1}{2}\right)\mathbf{a}_2 - \left(y_{13} - \tfrac{1}{2}\right)\mathbf{a}_3 \quad = \quad -az_{13}\,\hat{\mathbf{x}} + a\left(x_{13} + \tfrac{1}{2}\right)\hat{\mathbf{y}} - a\left(y_{13} - \tfrac{1}{2}\right)\hat{\mathbf{z}} \qquad \text{(24d)} \qquad \text{O VI}$$

$$\mathbf{B_{209}} = \quad y_{13}\,\mathbf{a}_1 + z_{13}\,\mathbf{a}_2 + x_{13}\,\mathbf{a}_3 \quad = \quad ay_{13}\,\hat{\mathbf{x}} + az_{13}\,\hat{\mathbf{y}} + ax_{13}\,\hat{\mathbf{z}} \qquad \text{(24d)} \qquad \text{O VI}$$

$$\mathbf{B_{210}} = \quad -y_{13}\,\mathbf{a}_1 + \left(z_{13} + \tfrac{1}{2}\right)\mathbf{a}_2 - \left(x_{13} - \tfrac{1}{2}\right)\mathbf{a}_3 \quad = \quad -ay_{13}\,\hat{\mathbf{x}} + a\left(z_{13} + \tfrac{1}{2}\right)\hat{\mathbf{y}} - a\left(x_{13} - \tfrac{1}{2}\right)\hat{\mathbf{z}} \qquad \text{(24d)} \qquad \text{O VI}$$

$$\mathbf{B_{211}} = \quad \left(y_{13} + \tfrac{1}{2}\right)\mathbf{a}_1 - \left(z_{13} - \tfrac{1}{2}\right)\mathbf{a}_2 - x_{13}\,\mathbf{a}_3 \quad = \quad a\left(y_{13} + \tfrac{1}{2}\right)\hat{\mathbf{x}} - a\left(z_{13} - \tfrac{1}{2}\right)\hat{\mathbf{y}} - ax_{13}\,\hat{\mathbf{z}} \qquad \text{(24d)} \qquad \text{O VI}$$

$$\mathbf{B_{212}} = -\left(y_{13} - \tfrac{1}{2}\right)\mathbf{a}_1 - z_{13}\,\mathbf{a}_2 + \left(x_{13} + \tfrac{1}{2}\right)\mathbf{a}_3 \qquad = \qquad -a\left(y_{13} - \tfrac{1}{2}\right)\hat{\mathbf{x}} - az_{13}\,\hat{\mathbf{y}} + a\left(x_{13} + \tfrac{1}{2}\right)\hat{\mathbf{z}} \qquad (24\text{d}) \qquad \text{O VI}$$

$$\mathbf{B_{213}} = -x_{13}\,\mathbf{a}_1 - y_{13}\,\mathbf{a}_2 - z_{13}\,\mathbf{a}_3 \qquad = \qquad -ax_{13}\,\hat{\mathbf{x}} - ay_{13}\,\hat{\mathbf{y}} - az_{13}\,\hat{\mathbf{z}} \qquad (24\text{d}) \qquad \text{O VI}$$

$$\mathbf{B_{214}} = \left(x_{13} + \tfrac{1}{2}\right)\mathbf{a}_1 + y_{13}\,\mathbf{a}_2 - \left(z_{13} - \tfrac{1}{2}\right)\mathbf{a}_3 \qquad = \qquad a\left(x_{13} + \tfrac{1}{2}\right)\hat{\mathbf{x}} + ay_{13}\,\hat{\mathbf{y}} - a\left(z_{13} - \tfrac{1}{2}\right)\hat{\mathbf{z}} \qquad (24\text{d}) \qquad \text{O VI}$$

$$\mathbf{B_{215}} = x_{13}\,\mathbf{a}_1 - \left(y_{13} - \tfrac{1}{2}\right)\mathbf{a}_2 + \left(z_{13} + \tfrac{1}{2}\right)\mathbf{a}_3 \qquad = \qquad ax_{13}\,\hat{\mathbf{x}} - a\left(y_{13} - \tfrac{1}{2}\right)\hat{\mathbf{y}} + a\left(z_{13} + \tfrac{1}{2}\right)\hat{\mathbf{z}} \qquad (24\text{d}) \qquad \text{O VI}$$

$$\mathbf{B_{216}} = -\left(x_{13} - \tfrac{1}{2}\right)\mathbf{a}_1 + \left(y_{13} + \tfrac{1}{2}\right)\mathbf{a}_2 + z_{13}\,\mathbf{a}_3 \qquad = \qquad -a\left(x_{13} - \tfrac{1}{2}\right)\hat{\mathbf{x}} + a\left(y_{13} + \tfrac{1}{2}\right)\hat{\mathbf{y}} + az_{13}\,\hat{\mathbf{z}} \qquad (24\text{d}) \qquad \text{O VI}$$

$$\mathbf{B_{217}} = -z_{13}\,\mathbf{a}_1 - x_{13}\,\mathbf{a}_2 - y_{13}\,\mathbf{a}_3 \qquad = \qquad -az_{13}\,\hat{\mathbf{x}} - ax_{13}\,\hat{\mathbf{y}} - ay_{13}\,\hat{\mathbf{z}} \qquad (24\text{d}) \qquad \text{O VI}$$

$$\mathbf{B_{218}} = -\left(z_{13} - \tfrac{1}{2}\right)\mathbf{a}_1 + \left(x_{13} + \tfrac{1}{2}\right)\mathbf{a}_2 + y_{13}\,\mathbf{a}_3 \qquad = \qquad -a\left(z_{13} - \tfrac{1}{2}\right)\hat{\mathbf{x}} + a\left(x_{13} + \tfrac{1}{2}\right)\hat{\mathbf{y}} + ay_{13}\,\hat{\mathbf{z}} \qquad (24\text{d}) \qquad \text{O VI}$$

$$\mathbf{B_{219}} = \left(z_{13} + \tfrac{1}{2}\right)\mathbf{a}_1 + x_{13}\,\mathbf{a}_2 - \left(y_{13} - \tfrac{1}{2}\right)\mathbf{a}_3 \qquad = \qquad a\left(z_{13} + \tfrac{1}{2}\right)\hat{\mathbf{x}} + ax_{13}\,\hat{\mathbf{y}} - a\left(y_{13} - \tfrac{1}{2}\right)\hat{\mathbf{z}} \qquad (24\text{d}) \qquad \text{O VI}$$

$$\mathbf{B_{220}} = z_{13}\,\mathbf{a}_1 - \left(x_{13} - \tfrac{1}{2}\right)\mathbf{a}_2 + \left(y_{13} + \tfrac{1}{2}\right)\mathbf{a}_3 \qquad = \qquad az_{13}\,\hat{\mathbf{x}} - a\left(x_{13} - \tfrac{1}{2}\right)\hat{\mathbf{y}} + a\left(y_{13} + \tfrac{1}{2}\right)\hat{\mathbf{z}} \qquad (24\text{d}) \qquad \text{O VI}$$

$$\mathbf{B_{221}} = -y_{13}\,\mathbf{a}_1 - z_{13}\,\mathbf{a}_2 - x_{13}\,\mathbf{a}_3 \qquad = \qquad -ay_{13}\,\hat{\mathbf{x}} - az_{13}\,\hat{\mathbf{y}} - ax_{13}\,\hat{\mathbf{z}} \qquad (24\text{d}) \qquad \text{O VI}$$

$$\mathbf{B_{222}} = y_{13}\,\mathbf{a}_1 - \left(z_{13} - \tfrac{1}{2}\right)\mathbf{a}_2 + \left(x_{13} + \tfrac{1}{2}\right)\mathbf{a}_3 \qquad = \qquad ay_{13}\,\hat{\mathbf{x}} - a\left(z_{13} - \tfrac{1}{2}\right)\hat{\mathbf{y}} + a\left(x_{13} + \tfrac{1}{2}\right)\hat{\mathbf{z}} \qquad (24\text{d}) \qquad \text{O VI}$$

$$\mathbf{B_{223}} = -\left(y_{13} - \tfrac{1}{2}\right)\mathbf{a}_1 + \left(z_{13} + \tfrac{1}{2}\right)\mathbf{a}_2 + x_{13}\,\mathbf{a}_3 \qquad = \qquad -a\left(y_{13} - \tfrac{1}{2}\right)\hat{\mathbf{x}} + a\left(z_{13} + \tfrac{1}{2}\right)\hat{\mathbf{y}} + ax_{13}\,\hat{\mathbf{z}} \qquad (24\text{d}) \qquad \text{O VI}$$

$$\mathbf{B_{224}} = \left(y_{13} + \tfrac{1}{2}\right)\mathbf{a}_1 + z_{13}\,\mathbf{a}_2 - \left(x_{13} - \tfrac{1}{2}\right)\mathbf{a}_3 \qquad = \qquad a\left(y_{13} + \tfrac{1}{2}\right)\hat{\mathbf{x}} + az_{13}\,\hat{\mathbf{y}} - a\left(x_{13} - \tfrac{1}{2}\right)\hat{\mathbf{z}} \qquad (24\text{d}) \qquad \text{O VI}$$

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