

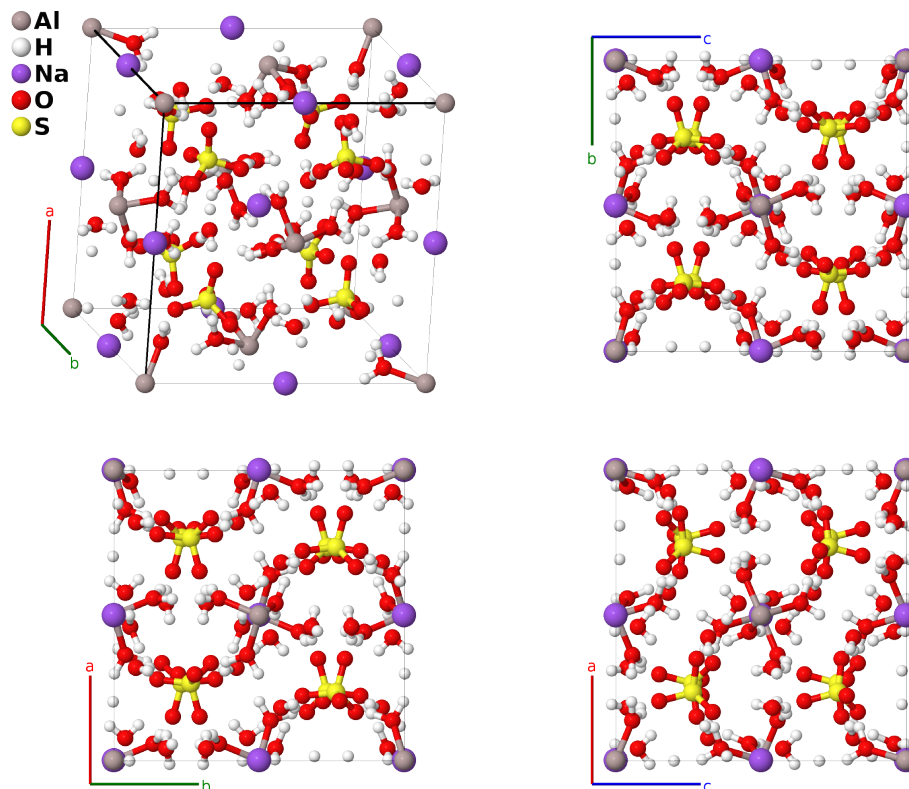
γ -Alum $[\text{AlNa}(\text{SO}_4)_2 \cdot 12\text{H}_2\text{O}, H4_{15}]$ Structure: AB24CD20E2_cP192_205_a_4d_b_c3d_c-001

This structure originally had the label AB24CD20E2_cP192_205_a_4d_b_c3d_c. Calls to that address will be redirected here.

Cite this page as: D. Hicks, M. J. Mehl, M. Esters, C. Oses, O. Levy, G. L. W. Hart, C. Toher, and S. Curtarolo, *The AFLOW Library of Crystallographic Prototypes: Part 3*, Comput. Mater. Sci. **199**, 110450 (2021), doi: 10.1016/j.commatsci.2021.110450.

<https://aflow.org/p/VBLW>

https://aflow.org/p/AB24CD20E2_cP192_205_a_4d_b_c3d_c-001



Prototype	$\text{AlH}_{24}\text{NaO}_{20}\text{S}_2$
AFLOW prototype label	AB24CD20E2_cP192_205_a_4d_b_c3d_c-001
<i>Strukturbericht</i> designation	$H4_{15}$
Mineral name	γ -alum
ICSD	15368
Pearson symbol	cP192
Space group number	205
Space group symbol	$Pa\bar{3}$
AFLOW prototype command	<pre>aflow --proto=AB24CD20E2_cP192_205_a_4d_b_c3d_c-001 --params=a, x3, x4, x5, y5, z5, x6, y6, z6, x7, y7, z7, x8, y8, z8, x9, y9, z9, x10, y10, z10, x11, y11, z11</pre>

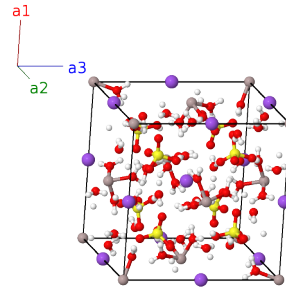
- The alums have the general formula $AB(XO_4)_2 \cdot 12H_2O$, where A is a monovalent ion, B is a trivalent ion, and X is a chalcogen. In most cases atom B is aluminum and atom X is sulfur, leading to the name alum.
- All alums have their room-temperature form in space group $Pa\bar{3}$ #205, but the bonding between the A and B ions and the XO_4 complex can be quite different.
- (Lipson, 1935ab) described three general forms of alum based on the sizes of the monovalent ions. Each of these forms was given a *Strukturbericht* designation by (Gottfried, 1937):
 - α -alum, with intermediate sized ions, prototype $KAl(SO_4)_2 \cdot 12H_2O$, $H4_{13}$,
 - β -alum, with large ions, prototype $(NH_3CH_3)Al(SO_4)_2 \cdot 12H_2O$, $H4_{14}$, and
 - γ -alum, with small ions, prototype $NaAl(SO_4)_2 \cdot 12H_2O$, $H4_{15}$ (this structure).

This classification scheme is not complete, *e.g.*, (Ledsham, 1968) points out that $NaCr(SO_4)_2 \cdot 12H_2O$ does not fit into any of these categories, and that the actual structure depends on the combination of monovalent and trivalent ions.

- As noted above, the $Pa\bar{3}$ structures of alum are the room temperature form. As the temperature decreases the alum structure may transform. For example, in the temperature range 150-170K the β -alum $(NH_3CH_3)Al(SO_4)_2 \cdot 12H_2O$ transforms into an orthorhombic structure with fully ordered NH_3CH_3 ions.

Simple Cubic primitive vectors

$$\begin{aligned} \mathbf{a}_1 &= a \hat{\mathbf{x}} \\ \mathbf{a}_2 &= a \hat{\mathbf{y}} \\ \mathbf{a}_3 &= a \hat{\mathbf{z}} \end{aligned}$$



Basis vectors

	Lattice coordinates		Cartesian coordinates	Wyckoff position	Atom type
\mathbf{B}_1	0	$=$	0	(4a)	Al I
\mathbf{B}_2	$\frac{1}{2} \mathbf{a}_1 + \frac{1}{2} \mathbf{a}_3$	$=$	$\frac{1}{2} a \hat{\mathbf{x}} + \frac{1}{2} a \hat{\mathbf{z}}$	(4a)	Al I
\mathbf{B}_3	$\frac{1}{2} \mathbf{a}_2 + \frac{1}{2} \mathbf{a}_3$	$=$	$\frac{1}{2} a \hat{\mathbf{y}} + \frac{1}{2} a \hat{\mathbf{z}}$	(4a)	Al I
\mathbf{B}_4	$\frac{1}{2} \mathbf{a}_1 + \frac{1}{2} \mathbf{a}_2$	$=$	$\frac{1}{2} a \hat{\mathbf{x}} + \frac{1}{2} a \hat{\mathbf{y}}$	(4a)	Al I
\mathbf{B}_5	$\frac{1}{2} \mathbf{a}_1 + \frac{1}{2} \mathbf{a}_2 + \frac{1}{2} \mathbf{a}_3$	$=$	$\frac{1}{2} a \hat{\mathbf{x}} + \frac{1}{2} a \hat{\mathbf{y}} + \frac{1}{2} a \hat{\mathbf{z}}$	(4b)	Na I
\mathbf{B}_6	$\frac{1}{2} \mathbf{a}_2$	$=$	$\frac{1}{2} a \hat{\mathbf{y}}$	(4b)	Na I
\mathbf{B}_7	$\frac{1}{2} \mathbf{a}_1$	$=$	$\frac{1}{2} a \hat{\mathbf{x}}$	(4b)	Na I
\mathbf{B}_8	$\frac{1}{2} \mathbf{a}_3$	$=$	$\frac{1}{2} a \hat{\mathbf{z}}$	(4b)	Na I
\mathbf{B}_9	$x_3 \mathbf{a}_1 + x_3 \mathbf{a}_2 + x_3 \mathbf{a}_3$	$=$	$ax_3 \hat{\mathbf{x}} + ax_3 \hat{\mathbf{y}} + ax_3 \hat{\mathbf{z}}$	(8c)	O I
\mathbf{B}_{10}	$-(x_3 - \frac{1}{2}) \mathbf{a}_1 - x_3 \mathbf{a}_2 + (x_3 + \frac{1}{2}) \mathbf{a}_3$	$=$	$-a(x_3 - \frac{1}{2}) \hat{\mathbf{x}} - ax_3 \hat{\mathbf{y}} + a(x_3 + \frac{1}{2}) \hat{\mathbf{z}}$	(8c)	O I
\mathbf{B}_{11}	$-x_3 \mathbf{a}_1 + (x_3 + \frac{1}{2}) \mathbf{a}_2 - (x_3 - \frac{1}{2}) \mathbf{a}_3$	$=$	$-ax_3 \hat{\mathbf{x}} + a(x_3 + \frac{1}{2}) \hat{\mathbf{y}} - a(x_3 - \frac{1}{2}) \hat{\mathbf{z}}$	(8c)	O I
\mathbf{B}_{12}	$(x_3 + \frac{1}{2}) \mathbf{a}_1 - (x_3 - \frac{1}{2}) \mathbf{a}_2 - x_3 \mathbf{a}_3$	$=$	$a(x_3 + \frac{1}{2}) \hat{\mathbf{x}} - a(x_3 - \frac{1}{2}) \hat{\mathbf{y}} - ax_3 \hat{\mathbf{z}}$	(8c)	O I

$$\begin{aligned}
\mathbf{B}_{13} &= -x_3 \mathbf{a}_1 - x_3 \mathbf{a}_2 - x_3 \mathbf{a}_3 &= -ax_3 \hat{\mathbf{x}} - ax_3 \hat{\mathbf{y}} - ax_3 \hat{\mathbf{z}} & (8c) & \text{O I} \\
\mathbf{B}_{14} &= (x_3 + \frac{1}{2}) \mathbf{a}_1 + x_3 \mathbf{a}_2 - (x_3 - \frac{1}{2}) \mathbf{a}_3 &= a(x_3 + \frac{1}{2}) \hat{\mathbf{x}} + ax_3 \hat{\mathbf{y}} - a(x_3 - \frac{1}{2}) \hat{\mathbf{z}} & (8c) & \text{O I} \\
\mathbf{B}_{15} &= x_3 \mathbf{a}_1 - (x_3 - \frac{1}{2}) \mathbf{a}_2 + (x_3 + \frac{1}{2}) \mathbf{a}_3 &= ax_3 \hat{\mathbf{x}} - a(x_3 - \frac{1}{2}) \hat{\mathbf{y}} + a(x_3 + \frac{1}{2}) \hat{\mathbf{z}} & (8c) & \text{O I} \\
\mathbf{B}_{16} &= -(x_3 - \frac{1}{2}) \mathbf{a}_1 + (x_3 + \frac{1}{2}) \mathbf{a}_2 + x_3 \mathbf{a}_3 &= -a(x_3 - \frac{1}{2}) \hat{\mathbf{x}} + a(x_3 + \frac{1}{2}) \hat{\mathbf{y}} + ax_3 \hat{\mathbf{z}} & (8c) & \text{O I} \\
\mathbf{B}_{17} &= x_4 \mathbf{a}_1 + x_4 \mathbf{a}_2 + x_4 \mathbf{a}_3 &= ax_4 \hat{\mathbf{x}} + ax_4 \hat{\mathbf{y}} + ax_4 \hat{\mathbf{z}} & (8c) & \text{S I} \\
\mathbf{B}_{18} &= -(x_4 - \frac{1}{2}) \mathbf{a}_1 - x_4 \mathbf{a}_2 + (x_4 + \frac{1}{2}) \mathbf{a}_3 &= -a(x_4 - \frac{1}{2}) \hat{\mathbf{x}} - ax_4 \hat{\mathbf{y}} + a(x_4 + \frac{1}{2}) \hat{\mathbf{z}} & (8c) & \text{S I} \\
\mathbf{B}_{19} &= -x_4 \mathbf{a}_1 + (x_4 + \frac{1}{2}) \mathbf{a}_2 - (x_4 - \frac{1}{2}) \mathbf{a}_3 &= -ax_4 \hat{\mathbf{x}} + a(x_4 + \frac{1}{2}) \hat{\mathbf{y}} - a(x_4 - \frac{1}{2}) \hat{\mathbf{z}} & (8c) & \text{S I} \\
\mathbf{B}_{20} &= (x_4 + \frac{1}{2}) \mathbf{a}_1 - (x_4 - \frac{1}{2}) \mathbf{a}_2 - x_4 \mathbf{a}_3 &= a(x_4 + \frac{1}{2}) \hat{\mathbf{x}} - a(x_4 - \frac{1}{2}) \hat{\mathbf{y}} - ax_4 \hat{\mathbf{z}} & (8c) & \text{S I} \\
\mathbf{B}_{21} &= -x_4 \mathbf{a}_1 - x_4 \mathbf{a}_2 - x_4 \mathbf{a}_3 &= -ax_4 \hat{\mathbf{x}} - ax_4 \hat{\mathbf{y}} - ax_4 \hat{\mathbf{z}} & (8c) & \text{S I} \\
\mathbf{B}_{22} &= (x_4 + \frac{1}{2}) \mathbf{a}_1 + x_4 \mathbf{a}_2 - (x_4 - \frac{1}{2}) \mathbf{a}_3 &= a(x_4 + \frac{1}{2}) \hat{\mathbf{x}} + ax_4 \hat{\mathbf{y}} - a(x_4 - \frac{1}{2}) \hat{\mathbf{z}} & (8c) & \text{S I} \\
\mathbf{B}_{23} &= x_4 \mathbf{a}_1 - (x_4 - \frac{1}{2}) \mathbf{a}_2 + (x_4 + \frac{1}{2}) \mathbf{a}_3 &= ax_4 \hat{\mathbf{x}} - a(x_4 - \frac{1}{2}) \hat{\mathbf{y}} + a(x_4 + \frac{1}{2}) \hat{\mathbf{z}} & (8c) & \text{S I} \\
\mathbf{B}_{24} &= -(x_4 - \frac{1}{2}) \mathbf{a}_1 + (x_4 + \frac{1}{2}) \mathbf{a}_2 + x_4 \mathbf{a}_3 &= -a(x_4 - \frac{1}{2}) \hat{\mathbf{x}} + a(x_4 + \frac{1}{2}) \hat{\mathbf{y}} + ax_4 \hat{\mathbf{z}} & (8c) & \text{S I} \\
\mathbf{B}_{25} &= x_5 \mathbf{a}_1 + y_5 \mathbf{a}_2 + z_5 \mathbf{a}_3 &= ax_5 \hat{\mathbf{x}} + ay_5 \hat{\mathbf{y}} + az_5 \hat{\mathbf{z}} & (24d) & \text{H I} \\
\mathbf{B}_{26} &= -(x_5 - \frac{1}{2}) \mathbf{a}_1 - y_5 \mathbf{a}_2 + (z_5 + \frac{1}{2}) \mathbf{a}_3 &= -a(x_5 - \frac{1}{2}) \hat{\mathbf{x}} - ay_5 \hat{\mathbf{y}} + a(z_5 + \frac{1}{2}) \hat{\mathbf{z}} & (24d) & \text{H I} \\
\mathbf{B}_{27} &= -x_5 \mathbf{a}_1 + (y_5 + \frac{1}{2}) \mathbf{a}_2 - (z_5 - \frac{1}{2}) \mathbf{a}_3 &= -ax_5 \hat{\mathbf{x}} + a(y_5 + \frac{1}{2}) \hat{\mathbf{y}} - a(z_5 - \frac{1}{2}) \hat{\mathbf{z}} & (24d) & \text{H I} \\
\mathbf{B}_{28} &= (x_5 + \frac{1}{2}) \mathbf{a}_1 - (y_5 - \frac{1}{2}) \mathbf{a}_2 - z_5 \mathbf{a}_3 &= a(x_5 + \frac{1}{2}) \hat{\mathbf{x}} - a(y_5 - \frac{1}{2}) \hat{\mathbf{y}} - az_5 \hat{\mathbf{z}} & (24d) & \text{H I} \\
\mathbf{B}_{29} &= z_5 \mathbf{a}_1 + x_5 \mathbf{a}_2 + y_5 \mathbf{a}_3 &= az_5 \hat{\mathbf{x}} + ax_5 \hat{\mathbf{y}} + ay_5 \hat{\mathbf{z}} & (24d) & \text{H I} \\
\mathbf{B}_{30} &= (z_5 + \frac{1}{2}) \mathbf{a}_1 - (x_5 - \frac{1}{2}) \mathbf{a}_2 - y_5 \mathbf{a}_3 &= a(z_5 + \frac{1}{2}) \hat{\mathbf{x}} - a(x_5 - \frac{1}{2}) \hat{\mathbf{y}} - ay_5 \hat{\mathbf{z}} & (24d) & \text{H I} \\
\mathbf{B}_{31} &= -(z_5 - \frac{1}{2}) \mathbf{a}_1 - x_5 \mathbf{a}_2 + (y_5 + \frac{1}{2}) \mathbf{a}_3 &= -a(z_5 - \frac{1}{2}) \hat{\mathbf{x}} - ax_5 \hat{\mathbf{y}} + a(y_5 + \frac{1}{2}) \hat{\mathbf{z}} & (24d) & \text{H I} \\
\mathbf{B}_{32} &= -z_5 \mathbf{a}_1 + (x_5 + \frac{1}{2}) \mathbf{a}_2 - (y_5 - \frac{1}{2}) \mathbf{a}_3 &= -az_5 \hat{\mathbf{x}} + a(x_5 + \frac{1}{2}) \hat{\mathbf{y}} - a(y_5 - \frac{1}{2}) \hat{\mathbf{z}} & (24d) & \text{H I} \\
\mathbf{B}_{33} &= y_5 \mathbf{a}_1 + z_5 \mathbf{a}_2 + x_5 \mathbf{a}_3 &= ay_5 \hat{\mathbf{x}} + az_5 \hat{\mathbf{y}} + ax_5 \hat{\mathbf{z}} & (24d) & \text{H I} \\
\mathbf{B}_{34} &= -y_5 \mathbf{a}_1 + (z_5 + \frac{1}{2}) \mathbf{a}_2 - (x_5 - \frac{1}{2}) \mathbf{a}_3 &= -ay_5 \hat{\mathbf{x}} + a(z_5 + \frac{1}{2}) \hat{\mathbf{y}} - a(x_5 - \frac{1}{2}) \hat{\mathbf{z}} & (24d) & \text{H I} \\
\mathbf{B}_{35} &= (y_5 + \frac{1}{2}) \mathbf{a}_1 - (z_5 - \frac{1}{2}) \mathbf{a}_2 - x_5 \mathbf{a}_3 &= a(y_5 + \frac{1}{2}) \hat{\mathbf{x}} - a(z_5 - \frac{1}{2}) \hat{\mathbf{y}} - ax_5 \hat{\mathbf{z}} & (24d) & \text{H I} \\
\mathbf{B}_{36} &= -(y_5 - \frac{1}{2}) \mathbf{a}_1 - z_5 \mathbf{a}_2 + (x_5 + \frac{1}{2}) \mathbf{a}_3 &= -a(y_5 - \frac{1}{2}) \hat{\mathbf{x}} - az_5 \hat{\mathbf{y}} + a(x_5 + \frac{1}{2}) \hat{\mathbf{z}} & (24d) & \text{H I} \\
\mathbf{B}_{37} &= -x_5 \mathbf{a}_1 - y_5 \mathbf{a}_2 - z_5 \mathbf{a}_3 &= -ax_5 \hat{\mathbf{x}} - ay_5 \hat{\mathbf{y}} - az_5 \hat{\mathbf{z}} & (24d) & \text{H I} \\
\mathbf{B}_{38} &= (x_5 + \frac{1}{2}) \mathbf{a}_1 + y_5 \mathbf{a}_2 - (z_5 - \frac{1}{2}) \mathbf{a}_3 &= a(x_5 + \frac{1}{2}) \hat{\mathbf{x}} + ay_5 \hat{\mathbf{y}} - a(z_5 - \frac{1}{2}) \hat{\mathbf{z}} & (24d) & \text{H I} \\
\mathbf{B}_{39} &= x_5 \mathbf{a}_1 - (y_5 - \frac{1}{2}) \mathbf{a}_2 + (z_5 + \frac{1}{2}) \mathbf{a}_3 &= ax_5 \hat{\mathbf{x}} - a(y_5 - \frac{1}{2}) \hat{\mathbf{y}} + a(z_5 + \frac{1}{2}) \hat{\mathbf{z}} & (24d) & \text{H I} \\
\mathbf{B}_{40} &= -(x_5 - \frac{1}{2}) \mathbf{a}_1 + (y_5 + \frac{1}{2}) \mathbf{a}_2 + z_5 \mathbf{a}_3 &= -a(x_5 - \frac{1}{2}) \hat{\mathbf{x}} + a(y_5 + \frac{1}{2}) \hat{\mathbf{y}} + az_5 \hat{\mathbf{z}} & (24d) & \text{H I} \\
\mathbf{B}_{41} &= -z_5 \mathbf{a}_1 - x_5 \mathbf{a}_2 - y_5 \mathbf{a}_3 &= -az_5 \hat{\mathbf{x}} - ax_5 \hat{\mathbf{y}} - ay_5 \hat{\mathbf{z}} & (24d) & \text{H I} \\
\mathbf{B}_{42} &= -(z_5 - \frac{1}{2}) \mathbf{a}_1 + (x_5 + \frac{1}{2}) \mathbf{a}_2 + y_5 \mathbf{a}_3 &= -a(z_5 - \frac{1}{2}) \hat{\mathbf{x}} + a(x_5 + \frac{1}{2}) \hat{\mathbf{y}} + ay_5 \hat{\mathbf{z}} & (24d) & \text{H I} \\
\mathbf{B}_{43} &= (z_5 + \frac{1}{2}) \mathbf{a}_1 + x_5 \mathbf{a}_2 - (y_5 - \frac{1}{2}) \mathbf{a}_3 &= a(z_5 + \frac{1}{2}) \hat{\mathbf{x}} + ax_5 \hat{\mathbf{y}} - a(y_5 - \frac{1}{2}) \hat{\mathbf{z}} & (24d) & \text{H I} \\
\mathbf{B}_{44} &= z_5 \mathbf{a}_1 - (x_5 - \frac{1}{2}) \mathbf{a}_2 + (y_5 + \frac{1}{2}) \mathbf{a}_3 &= az_5 \hat{\mathbf{x}} - a(x_5 - \frac{1}{2}) \hat{\mathbf{y}} + a(y_5 + \frac{1}{2}) \hat{\mathbf{z}} & (24d) & \text{H I}
\end{aligned}$$

$$\begin{aligned}
\mathbf{B}_{45} &= -y_5 \mathbf{a}_1 - z_5 \mathbf{a}_2 - x_5 \mathbf{a}_3 = -ay_5 \hat{\mathbf{x}} - az_5 \hat{\mathbf{y}} - ax_5 \hat{\mathbf{z}} & (24d) & \text{H I} \\
\mathbf{B}_{46} &= y_5 \mathbf{a}_1 - (z_5 - \frac{1}{2}) \mathbf{a}_2 + (x_5 + \frac{1}{2}) \mathbf{a}_3 = ay_5 \hat{\mathbf{x}} - a(z_5 - \frac{1}{2}) \hat{\mathbf{y}} + a(x_5 + \frac{1}{2}) \hat{\mathbf{z}} & (24d) & \text{H I} \\
\mathbf{B}_{47} &= -(y_5 - \frac{1}{2}) \mathbf{a}_1 + (z_5 + \frac{1}{2}) \mathbf{a}_2 + x_5 \mathbf{a}_3 = -a(y_5 - \frac{1}{2}) \hat{\mathbf{x}} + a(z_5 + \frac{1}{2}) \hat{\mathbf{y}} + ax_5 \hat{\mathbf{z}} & (24d) & \text{H I} \\
\mathbf{B}_{48} &= (y_5 + \frac{1}{2}) \mathbf{a}_1 + z_5 \mathbf{a}_2 - (x_5 - \frac{1}{2}) \mathbf{a}_3 = a(y_5 + \frac{1}{2}) \hat{\mathbf{x}} + az_5 \hat{\mathbf{y}} - a(x_5 - \frac{1}{2}) \hat{\mathbf{z}} & (24d) & \text{H I} \\
\mathbf{B}_{49} &= x_6 \mathbf{a}_1 + y_6 \mathbf{a}_2 + z_6 \mathbf{a}_3 = ax_6 \hat{\mathbf{x}} + ay_6 \hat{\mathbf{y}} + az_6 \hat{\mathbf{z}} & (24d) & \text{H II} \\
\mathbf{B}_{50} &= -(x_6 - \frac{1}{2}) \mathbf{a}_1 - y_6 \mathbf{a}_2 + (z_6 + \frac{1}{2}) \mathbf{a}_3 = -a(x_6 - \frac{1}{2}) \hat{\mathbf{x}} - ay_6 \hat{\mathbf{y}} + a(z_6 + \frac{1}{2}) \hat{\mathbf{z}} & (24d) & \text{H II} \\
\mathbf{B}_{51} &= -x_6 \mathbf{a}_1 + (y_6 + \frac{1}{2}) \mathbf{a}_2 - (z_6 - \frac{1}{2}) \mathbf{a}_3 = -ax_6 \hat{\mathbf{x}} + a(y_6 + \frac{1}{2}) \hat{\mathbf{y}} - a(z_6 - \frac{1}{2}) \hat{\mathbf{z}} & (24d) & \text{H II} \\
\mathbf{B}_{52} &= (x_6 + \frac{1}{2}) \mathbf{a}_1 - (y_6 - \frac{1}{2}) \mathbf{a}_2 - z_6 \mathbf{a}_3 = a(x_6 + \frac{1}{2}) \hat{\mathbf{x}} - a(y_6 - \frac{1}{2}) \hat{\mathbf{y}} - az_6 \hat{\mathbf{z}} & (24d) & \text{H II} \\
\mathbf{B}_{53} &= z_6 \mathbf{a}_1 + x_6 \mathbf{a}_2 + y_6 \mathbf{a}_3 = az_6 \hat{\mathbf{x}} + ax_6 \hat{\mathbf{y}} + ay_6 \hat{\mathbf{z}} & (24d) & \text{H II} \\
\mathbf{B}_{54} &= (z_6 + \frac{1}{2}) \mathbf{a}_1 - (x_6 - \frac{1}{2}) \mathbf{a}_2 - y_6 \mathbf{a}_3 = a(z_6 + \frac{1}{2}) \hat{\mathbf{x}} - a(x_6 - \frac{1}{2}) \hat{\mathbf{y}} - ay_6 \hat{\mathbf{z}} & (24d) & \text{H II} \\
\mathbf{B}_{55} &= -(z_6 - \frac{1}{2}) \mathbf{a}_1 - x_6 \mathbf{a}_2 + (y_6 + \frac{1}{2}) \mathbf{a}_3 = -a(z_6 - \frac{1}{2}) \hat{\mathbf{x}} - ax_6 \hat{\mathbf{y}} + a(y_6 + \frac{1}{2}) \hat{\mathbf{z}} & (24d) & \text{H II} \\
\mathbf{B}_{56} &= -z_6 \mathbf{a}_1 + (x_6 + \frac{1}{2}) \mathbf{a}_2 - (y_6 - \frac{1}{2}) \mathbf{a}_3 = -az_6 \hat{\mathbf{x}} + a(x_6 + \frac{1}{2}) \hat{\mathbf{y}} - a(y_6 - \frac{1}{2}) \hat{\mathbf{z}} & (24d) & \text{H II} \\
\mathbf{B}_{57} &= y_6 \mathbf{a}_1 + z_6 \mathbf{a}_2 + x_6 \mathbf{a}_3 = ay_6 \hat{\mathbf{x}} + az_6 \hat{\mathbf{y}} + ax_6 \hat{\mathbf{z}} & (24d) & \text{H II} \\
\mathbf{B}_{58} &= -y_6 \mathbf{a}_1 + (z_6 + \frac{1}{2}) \mathbf{a}_2 - (x_6 - \frac{1}{2}) \mathbf{a}_3 = -ay_6 \hat{\mathbf{x}} + a(z_6 + \frac{1}{2}) \hat{\mathbf{y}} - a(x_6 - \frac{1}{2}) \hat{\mathbf{z}} & (24d) & \text{H II} \\
\mathbf{B}_{59} &= (y_6 + \frac{1}{2}) \mathbf{a}_1 - (z_6 - \frac{1}{2}) \mathbf{a}_2 - x_6 \mathbf{a}_3 = a(y_6 + \frac{1}{2}) \hat{\mathbf{x}} - a(z_6 - \frac{1}{2}) \hat{\mathbf{y}} - ax_6 \hat{\mathbf{z}} & (24d) & \text{H II} \\
\mathbf{B}_{60} &= -(y_6 - \frac{1}{2}) \mathbf{a}_1 - z_6 \mathbf{a}_2 + (x_6 + \frac{1}{2}) \mathbf{a}_3 = -a(y_6 - \frac{1}{2}) \hat{\mathbf{x}} - az_6 \hat{\mathbf{y}} + a(x_6 + \frac{1}{2}) \hat{\mathbf{z}} & (24d) & \text{H II} \\
\mathbf{B}_{61} &= -x_6 \mathbf{a}_1 - y_6 \mathbf{a}_2 - z_6 \mathbf{a}_3 = -ax_6 \hat{\mathbf{x}} - ay_6 \hat{\mathbf{y}} - az_6 \hat{\mathbf{z}} & (24d) & \text{H II} \\
\mathbf{B}_{62} &= (x_6 + \frac{1}{2}) \mathbf{a}_1 + y_6 \mathbf{a}_2 - (z_6 - \frac{1}{2}) \mathbf{a}_3 = a(x_6 + \frac{1}{2}) \hat{\mathbf{x}} + ay_6 \hat{\mathbf{y}} - a(z_6 - \frac{1}{2}) \hat{\mathbf{z}} & (24d) & \text{H II} \\
\mathbf{B}_{63} &= x_6 \mathbf{a}_1 - (y_6 - \frac{1}{2}) \mathbf{a}_2 + (z_6 + \frac{1}{2}) \mathbf{a}_3 = ax_6 \hat{\mathbf{x}} - a(y_6 - \frac{1}{2}) \hat{\mathbf{y}} + a(z_6 + \frac{1}{2}) \hat{\mathbf{z}} & (24d) & \text{H II} \\
\mathbf{B}_{64} &= -(x_6 - \frac{1}{2}) \mathbf{a}_1 + (y_6 + \frac{1}{2}) \mathbf{a}_2 + z_6 \mathbf{a}_3 = -a(x_6 - \frac{1}{2}) \hat{\mathbf{x}} + a(y_6 + \frac{1}{2}) \hat{\mathbf{y}} + az_6 \hat{\mathbf{z}} & (24d) & \text{H II} \\
\mathbf{B}_{65} &= -z_6 \mathbf{a}_1 - x_6 \mathbf{a}_2 - y_6 \mathbf{a}_3 = -az_6 \hat{\mathbf{x}} - ax_6 \hat{\mathbf{y}} - ay_6 \hat{\mathbf{z}} & (24d) & \text{H II} \\
\mathbf{B}_{66} &= -(z_6 - \frac{1}{2}) \mathbf{a}_1 + (x_6 + \frac{1}{2}) \mathbf{a}_2 + y_6 \mathbf{a}_3 = -a(z_6 - \frac{1}{2}) \hat{\mathbf{x}} + a(x_6 + \frac{1}{2}) \hat{\mathbf{y}} + ay_6 \hat{\mathbf{z}} & (24d) & \text{H II} \\
\mathbf{B}_{67} &= (z_6 + \frac{1}{2}) \mathbf{a}_1 + x_6 \mathbf{a}_2 - (y_6 - \frac{1}{2}) \mathbf{a}_3 = a(z_6 + \frac{1}{2}) \hat{\mathbf{x}} + ax_6 \hat{\mathbf{y}} - a(y_6 - \frac{1}{2}) \hat{\mathbf{z}} & (24d) & \text{H II} \\
\mathbf{B}_{68} &= z_6 \mathbf{a}_1 - (x_6 - \frac{1}{2}) \mathbf{a}_2 + (y_6 + \frac{1}{2}) \mathbf{a}_3 = az_6 \hat{\mathbf{x}} - a(x_6 - \frac{1}{2}) \hat{\mathbf{y}} + a(y_6 + \frac{1}{2}) \hat{\mathbf{z}} & (24d) & \text{H II} \\
\mathbf{B}_{69} &= -y_6 \mathbf{a}_1 - z_6 \mathbf{a}_2 - x_6 \mathbf{a}_3 = -ay_6 \hat{\mathbf{x}} - az_6 \hat{\mathbf{y}} - ax_6 \hat{\mathbf{z}} & (24d) & \text{H II} \\
\mathbf{B}_{70} &= y_6 \mathbf{a}_1 - (z_6 - \frac{1}{2}) \mathbf{a}_2 + (x_6 + \frac{1}{2}) \mathbf{a}_3 = ay_6 \hat{\mathbf{x}} - a(z_6 - \frac{1}{2}) \hat{\mathbf{y}} + a(x_6 + \frac{1}{2}) \hat{\mathbf{z}} & (24d) & \text{H II} \\
\mathbf{B}_{71} &= -(y_6 - \frac{1}{2}) \mathbf{a}_1 + (z_6 + \frac{1}{2}) \mathbf{a}_2 + x_6 \mathbf{a}_3 = -a(y_6 - \frac{1}{2}) \hat{\mathbf{x}} + a(z_6 + \frac{1}{2}) \hat{\mathbf{y}} + ax_6 \hat{\mathbf{z}} & (24d) & \text{H II} \\
\mathbf{B}_{72} &= (y_6 + \frac{1}{2}) \mathbf{a}_1 + z_6 \mathbf{a}_2 - (x_6 - \frac{1}{2}) \mathbf{a}_3 = a(y_6 + \frac{1}{2}) \hat{\mathbf{x}} + az_6 \hat{\mathbf{y}} - a(x_6 - \frac{1}{2}) \hat{\mathbf{z}} & (24d) & \text{H II} \\
\mathbf{B}_{73} &= x_7 \mathbf{a}_1 + y_7 \mathbf{a}_2 + z_7 \mathbf{a}_3 = ax_7 \hat{\mathbf{x}} + ay_7 \hat{\mathbf{y}} + az_7 \hat{\mathbf{z}} & (24d) & \text{H III} \\
\mathbf{B}_{74} &= -(x_7 - \frac{1}{2}) \mathbf{a}_1 - y_7 \mathbf{a}_2 + (z_7 + \frac{1}{2}) \mathbf{a}_3 = -a(x_7 - \frac{1}{2}) \hat{\mathbf{x}} - ay_7 \hat{\mathbf{y}} + a(z_7 + \frac{1}{2}) \hat{\mathbf{z}} & (24d) & \text{H III} \\
\mathbf{B}_{75} &= -x_7 \mathbf{a}_1 + (y_7 + \frac{1}{2}) \mathbf{a}_2 - (z_7 - \frac{1}{2}) \mathbf{a}_3 = -ax_7 \hat{\mathbf{x}} + a(y_7 + \frac{1}{2}) \hat{\mathbf{y}} - a(z_7 - \frac{1}{2}) \hat{\mathbf{z}} & (24d) & \text{H III} \\
\mathbf{B}_{76} &= (x_7 + \frac{1}{2}) \mathbf{a}_1 - (y_7 - \frac{1}{2}) \mathbf{a}_2 - z_7 \mathbf{a}_3 = a(x_7 + \frac{1}{2}) \hat{\mathbf{x}} - a(y_7 - \frac{1}{2}) \hat{\mathbf{y}} - az_7 \hat{\mathbf{z}} & (24d) & \text{H III}
\end{aligned}$$

$$\begin{aligned}
\mathbf{B}_{77} &= z_7 \mathbf{a}_1 + x_7 \mathbf{a}_2 + y_7 \mathbf{a}_3 &= & az_7 \hat{\mathbf{x}} + ax_7 \hat{\mathbf{y}} + ay_7 \hat{\mathbf{z}} & (24d) & \text{H III} \\
\mathbf{B}_{78} &= (z_7 + \frac{1}{2}) \mathbf{a}_1 - (x_7 - \frac{1}{2}) \mathbf{a}_2 - y_7 \mathbf{a}_3 &= & a(z_7 + \frac{1}{2}) \hat{\mathbf{x}} - a(x_7 - \frac{1}{2}) \hat{\mathbf{y}} - ay_7 \hat{\mathbf{z}} & (24d) & \text{H III} \\
\mathbf{B}_{79} &= -(z_7 - \frac{1}{2}) \mathbf{a}_1 - x_7 \mathbf{a}_2 + (y_7 + \frac{1}{2}) \mathbf{a}_3 &= & -a(z_7 - \frac{1}{2}) \hat{\mathbf{x}} - ax_7 \hat{\mathbf{y}} + a(y_7 + \frac{1}{2}) \hat{\mathbf{z}} & (24d) & \text{H III} \\
\mathbf{B}_{80} &= -z_7 \mathbf{a}_1 + (x_7 + \frac{1}{2}) \mathbf{a}_2 - (y_7 - \frac{1}{2}) \mathbf{a}_3 &= & -az_7 \hat{\mathbf{x}} + a(x_7 + \frac{1}{2}) \hat{\mathbf{y}} - a(y_7 - \frac{1}{2}) \hat{\mathbf{z}} & (24d) & \text{H III} \\
\mathbf{B}_{81} &= y_7 \mathbf{a}_1 + z_7 \mathbf{a}_2 + x_7 \mathbf{a}_3 &= & ay_7 \hat{\mathbf{x}} + az_7 \hat{\mathbf{y}} + ax_7 \hat{\mathbf{z}} & (24d) & \text{H III} \\
\mathbf{B}_{82} &= -y_7 \mathbf{a}_1 + (z_7 + \frac{1}{2}) \mathbf{a}_2 - (x_7 - \frac{1}{2}) \mathbf{a}_3 &= & -ay_7 \hat{\mathbf{x}} + a(z_7 + \frac{1}{2}) \hat{\mathbf{y}} - a(x_7 - \frac{1}{2}) \hat{\mathbf{z}} & (24d) & \text{H III} \\
\mathbf{B}_{83} &= (y_7 + \frac{1}{2}) \mathbf{a}_1 - (z_7 - \frac{1}{2}) \mathbf{a}_2 - x_7 \mathbf{a}_3 &= & a(y_7 + \frac{1}{2}) \hat{\mathbf{x}} - a(z_7 - \frac{1}{2}) \hat{\mathbf{y}} - ax_7 \hat{\mathbf{z}} & (24d) & \text{H III} \\
\mathbf{B}_{84} &= -(y_7 - \frac{1}{2}) \mathbf{a}_1 - z_7 \mathbf{a}_2 + (x_7 + \frac{1}{2}) \mathbf{a}_3 &= & -a(y_7 - \frac{1}{2}) \hat{\mathbf{x}} - az_7 \hat{\mathbf{y}} + a(x_7 + \frac{1}{2}) \hat{\mathbf{z}} & (24d) & \text{H III} \\
\mathbf{B}_{85} &= -x_7 \mathbf{a}_1 - y_7 \mathbf{a}_2 - z_7 \mathbf{a}_3 &= & -ax_7 \hat{\mathbf{x}} - ay_7 \hat{\mathbf{y}} - az_7 \hat{\mathbf{z}} & (24d) & \text{H III} \\
\mathbf{B}_{86} &= (x_7 + \frac{1}{2}) \mathbf{a}_1 + y_7 \mathbf{a}_2 - (z_7 - \frac{1}{2}) \mathbf{a}_3 &= & a(x_7 + \frac{1}{2}) \hat{\mathbf{x}} + ay_7 \hat{\mathbf{y}} - a(z_7 - \frac{1}{2}) \hat{\mathbf{z}} & (24d) & \text{H III} \\
\mathbf{B}_{87} &= x_7 \mathbf{a}_1 - (y_7 - \frac{1}{2}) \mathbf{a}_2 + (z_7 + \frac{1}{2}) \mathbf{a}_3 &= & ax_7 \hat{\mathbf{x}} - a(y_7 - \frac{1}{2}) \hat{\mathbf{y}} + a(z_7 + \frac{1}{2}) \hat{\mathbf{z}} & (24d) & \text{H III} \\
\mathbf{B}_{88} &= -(x_7 - \frac{1}{2}) \mathbf{a}_1 + (y_7 + \frac{1}{2}) \mathbf{a}_2 + z_7 \mathbf{a}_3 &= & -a(x_7 - \frac{1}{2}) \hat{\mathbf{x}} + a(y_7 + \frac{1}{2}) \hat{\mathbf{y}} + az_7 \hat{\mathbf{z}} & (24d) & \text{H III} \\
\mathbf{B}_{89} &= -z_7 \mathbf{a}_1 - x_7 \mathbf{a}_2 - y_7 \mathbf{a}_3 &= & -az_7 \hat{\mathbf{x}} - ax_7 \hat{\mathbf{y}} - ay_7 \hat{\mathbf{z}} & (24d) & \text{H III} \\
\mathbf{B}_{90} &= -(z_7 - \frac{1}{2}) \mathbf{a}_1 + (x_7 + \frac{1}{2}) \mathbf{a}_2 + y_7 \mathbf{a}_3 &= & -a(z_7 - \frac{1}{2}) \hat{\mathbf{x}} + a(x_7 + \frac{1}{2}) \hat{\mathbf{y}} + ay_7 \hat{\mathbf{z}} & (24d) & \text{H III} \\
\mathbf{B}_{91} &= (z_7 + \frac{1}{2}) \mathbf{a}_1 + x_7 \mathbf{a}_2 - (y_7 - \frac{1}{2}) \mathbf{a}_3 &= & a(z_7 + \frac{1}{2}) \hat{\mathbf{x}} + ax_7 \hat{\mathbf{y}} - a(y_7 - \frac{1}{2}) \hat{\mathbf{z}} & (24d) & \text{H III} \\
\mathbf{B}_{92} &= z_7 \mathbf{a}_1 - (x_7 - \frac{1}{2}) \mathbf{a}_2 + (y_7 + \frac{1}{2}) \mathbf{a}_3 &= & az_7 \hat{\mathbf{x}} - a(x_7 - \frac{1}{2}) \hat{\mathbf{y}} + a(y_7 + \frac{1}{2}) \hat{\mathbf{z}} & (24d) & \text{H III} \\
\mathbf{B}_{93} &= -y_7 \mathbf{a}_1 - z_7 \mathbf{a}_2 - x_7 \mathbf{a}_3 &= & -ay_7 \hat{\mathbf{x}} - az_7 \hat{\mathbf{y}} - ax_7 \hat{\mathbf{z}} & (24d) & \text{H III} \\
\mathbf{B}_{94} &= y_7 \mathbf{a}_1 - (z_7 - \frac{1}{2}) \mathbf{a}_2 + (x_7 + \frac{1}{2}) \mathbf{a}_3 &= & ay_7 \hat{\mathbf{x}} - a(z_7 - \frac{1}{2}) \hat{\mathbf{y}} + a(x_7 + \frac{1}{2}) \hat{\mathbf{z}} & (24d) & \text{H III} \\
\mathbf{B}_{95} &= -(y_7 - \frac{1}{2}) \mathbf{a}_1 + (z_7 + \frac{1}{2}) \mathbf{a}_2 + x_7 \mathbf{a}_3 &= & -a(y_7 - \frac{1}{2}) \hat{\mathbf{x}} + a(z_7 + \frac{1}{2}) \hat{\mathbf{y}} + ax_7 \hat{\mathbf{z}} & (24d) & \text{H III} \\
\mathbf{B}_{96} &= (y_7 + \frac{1}{2}) \mathbf{a}_1 + z_7 \mathbf{a}_2 - (x_7 - \frac{1}{2}) \mathbf{a}_3 &= & a(y_7 + \frac{1}{2}) \hat{\mathbf{x}} + az_7 \hat{\mathbf{y}} - a(x_7 - \frac{1}{2}) \hat{\mathbf{z}} & (24d) & \text{H III} \\
\mathbf{B}_{97} &= x_8 \mathbf{a}_1 + y_8 \mathbf{a}_2 + z_8 \mathbf{a}_3 &= & ax_8 \hat{\mathbf{x}} + ay_8 \hat{\mathbf{y}} + az_8 \hat{\mathbf{z}} & (24d) & \text{H IV} \\
\mathbf{B}_{98} &= -(x_8 - \frac{1}{2}) \mathbf{a}_1 - y_8 \mathbf{a}_2 + (z_8 + \frac{1}{2}) \mathbf{a}_3 &= & -a(x_8 - \frac{1}{2}) \hat{\mathbf{x}} - ay_8 \hat{\mathbf{y}} + a(z_8 + \frac{1}{2}) \hat{\mathbf{z}} & (24d) & \text{H IV} \\
\mathbf{B}_{99} &= -x_8 \mathbf{a}_1 + (y_8 + \frac{1}{2}) \mathbf{a}_2 - (z_8 - \frac{1}{2}) \mathbf{a}_3 &= & -ax_8 \hat{\mathbf{x}} + a(y_8 + \frac{1}{2}) \hat{\mathbf{y}} - a(z_8 - \frac{1}{2}) \hat{\mathbf{z}} & (24d) & \text{H IV} \\
\mathbf{B}_{100} &= (x_8 + \frac{1}{2}) \mathbf{a}_1 - (y_8 - \frac{1}{2}) \mathbf{a}_2 - z_8 \mathbf{a}_3 &= & a(x_8 + \frac{1}{2}) \hat{\mathbf{x}} - a(y_8 - \frac{1}{2}) \hat{\mathbf{y}} - az_8 \hat{\mathbf{z}} & (24d) & \text{H IV} \\
\mathbf{B}_{101} &= z_8 \mathbf{a}_1 + x_8 \mathbf{a}_2 + y_8 \mathbf{a}_3 &= & az_8 \hat{\mathbf{x}} + ax_8 \hat{\mathbf{y}} + ay_8 \hat{\mathbf{z}} & (24d) & \text{H IV} \\
\mathbf{B}_{102} &= (z_8 + \frac{1}{2}) \mathbf{a}_1 - (x_8 - \frac{1}{2}) \mathbf{a}_2 - y_8 \mathbf{a}_3 &= & a(z_8 + \frac{1}{2}) \hat{\mathbf{x}} - a(x_8 - \frac{1}{2}) \hat{\mathbf{y}} - ay_8 \hat{\mathbf{z}} & (24d) & \text{H IV} \\
\mathbf{B}_{103} &= -(z_8 - \frac{1}{2}) \mathbf{a}_1 - x_8 \mathbf{a}_2 + (y_8 + \frac{1}{2}) \mathbf{a}_3 &= & -a(z_8 - \frac{1}{2}) \hat{\mathbf{x}} - ax_8 \hat{\mathbf{y}} + a(y_8 + \frac{1}{2}) \hat{\mathbf{z}} & (24d) & \text{H IV} \\
\mathbf{B}_{104} &= -z_8 \mathbf{a}_1 + (x_8 + \frac{1}{2}) \mathbf{a}_2 - (y_8 - \frac{1}{2}) \mathbf{a}_3 &= & -az_8 \hat{\mathbf{x}} + a(x_8 + \frac{1}{2}) \hat{\mathbf{y}} - a(y_8 - \frac{1}{2}) \hat{\mathbf{z}} & (24d) & \text{H IV} \\
\mathbf{B}_{105} &= y_8 \mathbf{a}_1 + z_8 \mathbf{a}_2 + x_8 \mathbf{a}_3 &= & ay_8 \hat{\mathbf{x}} + az_8 \hat{\mathbf{y}} + ax_8 \hat{\mathbf{z}} & (24d) & \text{H IV} \\
\mathbf{B}_{106} &= -y_8 \mathbf{a}_1 + (z_8 + \frac{1}{2}) \mathbf{a}_2 - (x_8 - \frac{1}{2}) \mathbf{a}_3 &= & -ay_8 \hat{\mathbf{x}} + a(z_8 + \frac{1}{2}) \hat{\mathbf{y}} - a(x_8 - \frac{1}{2}) \hat{\mathbf{z}} & (24d) & \text{H IV} \\
\mathbf{B}_{107} &= (y_8 + \frac{1}{2}) \mathbf{a}_1 - (z_8 - \frac{1}{2}) \mathbf{a}_2 - x_8 \mathbf{a}_3 &= & a(y_8 + \frac{1}{2}) \hat{\mathbf{x}} - a(z_8 - \frac{1}{2}) \hat{\mathbf{y}} - ax_8 \hat{\mathbf{z}} & (24d) & \text{H IV}
\end{aligned}$$

$$\begin{aligned}
\mathbf{B}_{140} &= z_9 \mathbf{a}_1 - (x_9 - \frac{1}{2}) \mathbf{a}_2 + (y_9 + \frac{1}{2}) \mathbf{a}_3 = az_9 \hat{\mathbf{x}} - a(x_9 - \frac{1}{2}) \hat{\mathbf{y}} + a(y_9 + \frac{1}{2}) \hat{\mathbf{z}} & (24d) & \text{O II} \\
\mathbf{B}_{141} &= -y_9 \mathbf{a}_1 - z_9 \mathbf{a}_2 - x_9 \mathbf{a}_3 = -ay_9 \hat{\mathbf{x}} - az_9 \hat{\mathbf{y}} - ax_9 \hat{\mathbf{z}} & (24d) & \text{O II} \\
\mathbf{B}_{142} &= y_9 \mathbf{a}_1 - (z_9 - \frac{1}{2}) \mathbf{a}_2 + (x_9 + \frac{1}{2}) \mathbf{a}_3 = ay_9 \hat{\mathbf{x}} - a(z_9 - \frac{1}{2}) \hat{\mathbf{y}} + a(x_9 + \frac{1}{2}) \hat{\mathbf{z}} & (24d) & \text{O II} \\
\mathbf{B}_{143} &= -(y_9 - \frac{1}{2}) \mathbf{a}_1 + (z_9 + \frac{1}{2}) \mathbf{a}_2 + x_9 \mathbf{a}_3 = -a(y_9 - \frac{1}{2}) \hat{\mathbf{x}} + a(z_9 + \frac{1}{2}) \hat{\mathbf{y}} + ax_9 \hat{\mathbf{z}} & (24d) & \text{O II} \\
\mathbf{B}_{144} &= (y_9 + \frac{1}{2}) \mathbf{a}_1 + z_9 \mathbf{a}_2 - (x_9 - \frac{1}{2}) \mathbf{a}_3 = a(y_9 + \frac{1}{2}) \hat{\mathbf{x}} + az_9 \hat{\mathbf{y}} - a(x_9 - \frac{1}{2}) \hat{\mathbf{z}} & (24d) & \text{O II} \\
\mathbf{B}_{145} &= x_{10} \mathbf{a}_1 + y_{10} \mathbf{a}_2 + z_{10} \mathbf{a}_3 = ax_{10} \hat{\mathbf{x}} + ay_{10} \hat{\mathbf{y}} + az_{10} \hat{\mathbf{z}} & (24d) & \text{O III} \\
\mathbf{B}_{146} &= -(x_{10} - \frac{1}{2}) \mathbf{a}_1 - y_{10} \mathbf{a}_2 + (z_{10} + \frac{1}{2}) \mathbf{a}_3 = -a(x_{10} - \frac{1}{2}) \hat{\mathbf{x}} - ay_{10} \hat{\mathbf{y}} + a(z_{10} + \frac{1}{2}) \hat{\mathbf{z}} & (24d) & \text{O III} \\
\mathbf{B}_{147} &= -x_{10} \mathbf{a}_1 + (y_{10} + \frac{1}{2}) \mathbf{a}_2 - (z_{10} - \frac{1}{2}) \mathbf{a}_3 = -ax_{10} \hat{\mathbf{x}} + a(y_{10} + \frac{1}{2}) \hat{\mathbf{y}} - a(z_{10} - \frac{1}{2}) \hat{\mathbf{z}} & (24d) & \text{O III} \\
\mathbf{B}_{148} &= (x_{10} + \frac{1}{2}) \mathbf{a}_1 - (y_{10} - \frac{1}{2}) \mathbf{a}_2 - z_{10} \mathbf{a}_3 = a(x_{10} + \frac{1}{2}) \hat{\mathbf{x}} - a(y_{10} - \frac{1}{2}) \hat{\mathbf{y}} - az_{10} \hat{\mathbf{z}} & (24d) & \text{O III} \\
\mathbf{B}_{149} &= z_{10} \mathbf{a}_1 + x_{10} \mathbf{a}_2 + y_{10} \mathbf{a}_3 = az_{10} \hat{\mathbf{x}} + ax_{10} \hat{\mathbf{y}} + ay_{10} \hat{\mathbf{z}} & (24d) & \text{O III} \\
\mathbf{B}_{150} &= (z_{10} + \frac{1}{2}) \mathbf{a}_1 - (x_{10} - \frac{1}{2}) \mathbf{a}_2 - y_{10} \mathbf{a}_3 = a(z_{10} + \frac{1}{2}) \hat{\mathbf{x}} - a(x_{10} - \frac{1}{2}) \hat{\mathbf{y}} - ay_{10} \hat{\mathbf{z}} & (24d) & \text{O III} \\
\mathbf{B}_{151} &= -(z_{10} - \frac{1}{2}) \mathbf{a}_1 - x_{10} \mathbf{a}_2 + (y_{10} + \frac{1}{2}) \mathbf{a}_3 = -a(z_{10} - \frac{1}{2}) \hat{\mathbf{x}} - ax_{10} \hat{\mathbf{y}} + a(y_{10} + \frac{1}{2}) \hat{\mathbf{z}} & (24d) & \text{O III} \\
\mathbf{B}_{152} &= -z_{10} \mathbf{a}_1 + (x_{10} + \frac{1}{2}) \mathbf{a}_2 - (y_{10} - \frac{1}{2}) \mathbf{a}_3 = -az_{10} \hat{\mathbf{x}} + a(x_{10} + \frac{1}{2}) \hat{\mathbf{y}} - a(y_{10} - \frac{1}{2}) \hat{\mathbf{z}} & (24d) & \text{O III} \\
\mathbf{B}_{153} &= y_{10} \mathbf{a}_1 + z_{10} \mathbf{a}_2 + x_{10} \mathbf{a}_3 = ay_{10} \hat{\mathbf{x}} + az_{10} \hat{\mathbf{y}} + ax_{10} \hat{\mathbf{z}} & (24d) & \text{O III} \\
\mathbf{B}_{154} &= -y_{10} \mathbf{a}_1 + (z_{10} + \frac{1}{2}) \mathbf{a}_2 - (x_{10} - \frac{1}{2}) \mathbf{a}_3 = -ay_{10} \hat{\mathbf{x}} + a(z_{10} + \frac{1}{2}) \hat{\mathbf{y}} - a(x_{10} - \frac{1}{2}) \hat{\mathbf{z}} & (24d) & \text{O III} \\
\mathbf{B}_{155} &= (y_{10} + \frac{1}{2}) \mathbf{a}_1 - (z_{10} - \frac{1}{2}) \mathbf{a}_2 - x_{10} \mathbf{a}_3 = a(y_{10} + \frac{1}{2}) \hat{\mathbf{x}} - a(z_{10} - \frac{1}{2}) \hat{\mathbf{y}} - ax_{10} \hat{\mathbf{z}} & (24d) & \text{O III} \\
\mathbf{B}_{156} &= -(y_{10} - \frac{1}{2}) \mathbf{a}_1 - z_{10} \mathbf{a}_2 + (x_{10} + \frac{1}{2}) \mathbf{a}_3 = -a(y_{10} - \frac{1}{2}) \hat{\mathbf{x}} - az_{10} \hat{\mathbf{y}} + a(x_{10} + \frac{1}{2}) \hat{\mathbf{z}} & (24d) & \text{O III} \\
\mathbf{B}_{157} &= -x_{10} \mathbf{a}_1 - y_{10} \mathbf{a}_2 - z_{10} \mathbf{a}_3 = -ax_{10} \hat{\mathbf{x}} - ay_{10} \hat{\mathbf{y}} - az_{10} \hat{\mathbf{z}} & (24d) & \text{O III} \\
\mathbf{B}_{158} &= (x_{10} + \frac{1}{2}) \mathbf{a}_1 + y_{10} \mathbf{a}_2 - (z_{10} - \frac{1}{2}) \mathbf{a}_3 = a(x_{10} + \frac{1}{2}) \hat{\mathbf{x}} + ay_{10} \hat{\mathbf{y}} - a(z_{10} - \frac{1}{2}) \hat{\mathbf{z}} & (24d) & \text{O III} \\
\mathbf{B}_{159} &= x_{10} \mathbf{a}_1 - (y_{10} - \frac{1}{2}) \mathbf{a}_2 + (z_{10} + \frac{1}{2}) \mathbf{a}_3 = ax_{10} \hat{\mathbf{x}} - a(y_{10} - \frac{1}{2}) \hat{\mathbf{y}} + a(z_{10} + \frac{1}{2}) \hat{\mathbf{z}} & (24d) & \text{O III} \\
\mathbf{B}_{160} &= -(x_{10} - \frac{1}{2}) \mathbf{a}_1 + (y_{10} + \frac{1}{2}) \mathbf{a}_2 + z_{10} \mathbf{a}_3 = -a(x_{10} - \frac{1}{2}) \hat{\mathbf{x}} + a(y_{10} + \frac{1}{2}) \hat{\mathbf{y}} + az_{10} \hat{\mathbf{z}} & (24d) & \text{O III} \\
\mathbf{B}_{161} &= -z_{10} \mathbf{a}_1 - x_{10} \mathbf{a}_2 - y_{10} \mathbf{a}_3 = -az_{10} \hat{\mathbf{x}} - ax_{10} \hat{\mathbf{y}} - ay_{10} \hat{\mathbf{z}} & (24d) & \text{O III} \\
\mathbf{B}_{162} &= -(z_{10} - \frac{1}{2}) \mathbf{a}_1 + (x_{10} + \frac{1}{2}) \mathbf{a}_2 + y_{10} \mathbf{a}_3 = -a(z_{10} - \frac{1}{2}) \hat{\mathbf{x}} + a(x_{10} + \frac{1}{2}) \hat{\mathbf{y}} + ay_{10} \hat{\mathbf{z}} & (24d) & \text{O III} \\
\mathbf{B}_{163} &= (z_{10} + \frac{1}{2}) \mathbf{a}_1 + x_{10} \mathbf{a}_2 - (y_{10} - \frac{1}{2}) \mathbf{a}_3 = a(z_{10} + \frac{1}{2}) \hat{\mathbf{x}} + ax_{10} \hat{\mathbf{y}} - a(y_{10} - \frac{1}{2}) \hat{\mathbf{z}} & (24d) & \text{O III} \\
\mathbf{B}_{164} &= z_{10} \mathbf{a}_1 - (x_{10} - \frac{1}{2}) \mathbf{a}_2 + (y_{10} + \frac{1}{2}) \mathbf{a}_3 = az_{10} \hat{\mathbf{x}} - a(x_{10} - \frac{1}{2}) \hat{\mathbf{y}} + a(y_{10} + \frac{1}{2}) \hat{\mathbf{z}} & (24d) & \text{O III} \\
\mathbf{B}_{165} &= -y_{10} \mathbf{a}_1 - z_{10} \mathbf{a}_2 - x_{10} \mathbf{a}_3 = -ay_{10} \hat{\mathbf{x}} - az_{10} \hat{\mathbf{y}} - ax_{10} \hat{\mathbf{z}} & (24d) & \text{O III} \\
\mathbf{B}_{166} &= y_{10} \mathbf{a}_1 - (z_{10} - \frac{1}{2}) \mathbf{a}_2 + (x_{10} + \frac{1}{2}) \mathbf{a}_3 = ay_{10} \hat{\mathbf{x}} - a(z_{10} - \frac{1}{2}) \hat{\mathbf{y}} + a(x_{10} + \frac{1}{2}) \hat{\mathbf{z}} & (24d) & \text{O III} \\
\mathbf{B}_{167} &= -(y_{10} - \frac{1}{2}) \mathbf{a}_1 + (z_{10} + \frac{1}{2}) \mathbf{a}_2 + x_{10} \mathbf{a}_3 = -a(y_{10} - \frac{1}{2}) \hat{\mathbf{x}} + a(z_{10} + \frac{1}{2}) \hat{\mathbf{y}} + ax_{10} \hat{\mathbf{z}} & (24d) & \text{O III}
\end{aligned}$$

$$\begin{aligned}
\mathbf{B}_{168} &= \begin{pmatrix} (y_{10} + \frac{1}{2}) \mathbf{a}_1 + z_{10} \mathbf{a}_2 - \\ (x_{10} - \frac{1}{2}) \mathbf{a}_3 \end{pmatrix} = a (y_{10} + \frac{1}{2}) \hat{\mathbf{x}} + az_{10} \hat{\mathbf{y}} - a (x_{10} - \frac{1}{2}) \hat{\mathbf{z}} & (24d) & \text{O III} \\
\mathbf{B}_{169} &= x_{11} \mathbf{a}_1 + y_{11} \mathbf{a}_2 + z_{11} \mathbf{a}_3 = ax_{11} \hat{\mathbf{x}} + ay_{11} \hat{\mathbf{y}} + az_{11} \hat{\mathbf{z}} & (24d) & \text{O IV} \\
\mathbf{B}_{170} &= - (x_{11} - \frac{1}{2}) \mathbf{a}_1 - y_{11} \mathbf{a}_2 + \\ & \quad (z_{11} + \frac{1}{2}) \mathbf{a}_3 = -a (x_{11} - \frac{1}{2}) \hat{\mathbf{x}} - ay_{11} \hat{\mathbf{y}} + a (z_{11} + \frac{1}{2}) \hat{\mathbf{z}} & (24d) & \text{O IV} \\
\mathbf{B}_{171} &= -x_{11} \mathbf{a}_1 + (y_{11} + \frac{1}{2}) \mathbf{a}_2 - \\ & \quad (z_{11} - \frac{1}{2}) \mathbf{a}_3 = -ax_{11} \hat{\mathbf{x}} + a (y_{11} + \frac{1}{2}) \hat{\mathbf{y}} - a (z_{11} - \frac{1}{2}) \hat{\mathbf{z}} & (24d) & \text{O IV} \\
\mathbf{B}_{172} &= (x_{11} + \frac{1}{2}) \mathbf{a}_1 - (y_{11} - \frac{1}{2}) \mathbf{a}_2 - \\ & \quad z_{11} \mathbf{a}_3 = a (x_{11} + \frac{1}{2}) \hat{\mathbf{x}} - a (y_{11} - \frac{1}{2}) \hat{\mathbf{y}} - az_{11} \hat{\mathbf{z}} & (24d) & \text{O IV} \\
\mathbf{B}_{173} &= z_{11} \mathbf{a}_1 + x_{11} \mathbf{a}_2 + y_{11} \mathbf{a}_3 = az_{11} \hat{\mathbf{x}} + ax_{11} \hat{\mathbf{y}} + ay_{11} \hat{\mathbf{z}} & (24d) & \text{O IV} \\
\mathbf{B}_{174} &= (z_{11} + \frac{1}{2}) \mathbf{a}_1 - (x_{11} - \frac{1}{2}) \mathbf{a}_2 - \\ & \quad y_{11} \mathbf{a}_3 = a (z_{11} + \frac{1}{2}) \hat{\mathbf{x}} - a (x_{11} - \frac{1}{2}) \hat{\mathbf{y}} - ay_{11} \hat{\mathbf{z}} & (24d) & \text{O IV} \\
\mathbf{B}_{175} &= - (z_{11} - \frac{1}{2}) \mathbf{a}_1 - x_{11} \mathbf{a}_2 + \\ & \quad (y_{11} + \frac{1}{2}) \mathbf{a}_3 = -a (z_{11} - \frac{1}{2}) \hat{\mathbf{x}} - ax_{11} \hat{\mathbf{y}} + a (y_{11} + \frac{1}{2}) \hat{\mathbf{z}} & (24d) & \text{O IV} \\
\mathbf{B}_{176} &= -z_{11} \mathbf{a}_1 + (x_{11} + \frac{1}{2}) \mathbf{a}_2 - \\ & \quad (y_{11} - \frac{1}{2}) \mathbf{a}_3 = -az_{11} \hat{\mathbf{x}} + a (x_{11} + \frac{1}{2}) \hat{\mathbf{y}} - a (y_{11} - \frac{1}{2}) \hat{\mathbf{z}} & (24d) & \text{O IV} \\
\mathbf{B}_{177} &= y_{11} \mathbf{a}_1 + z_{11} \mathbf{a}_2 + x_{11} \mathbf{a}_3 = ay_{11} \hat{\mathbf{x}} + az_{11} \hat{\mathbf{y}} + ax_{11} \hat{\mathbf{z}} & (24d) & \text{O IV} \\
\mathbf{B}_{178} &= -y_{11} \mathbf{a}_1 + (z_{11} + \frac{1}{2}) \mathbf{a}_2 - \\ & \quad (x_{11} - \frac{1}{2}) \mathbf{a}_3 = -ay_{11} \hat{\mathbf{x}} + a (z_{11} + \frac{1}{2}) \hat{\mathbf{y}} - a (x_{11} - \frac{1}{2}) \hat{\mathbf{z}} & (24d) & \text{O IV} \\
\mathbf{B}_{179} &= (y_{11} + \frac{1}{2}) \mathbf{a}_1 - (z_{11} - \frac{1}{2}) \mathbf{a}_2 - \\ & \quad x_{11} \mathbf{a}_3 = a (y_{11} + \frac{1}{2}) \hat{\mathbf{x}} - a (z_{11} - \frac{1}{2}) \hat{\mathbf{y}} - ax_{11} \hat{\mathbf{z}} & (24d) & \text{O IV} \\
\mathbf{B}_{180} &= - (y_{11} - \frac{1}{2}) \mathbf{a}_1 - z_{11} \mathbf{a}_2 + \\ & \quad (x_{11} + \frac{1}{2}) \mathbf{a}_3 = -a (y_{11} - \frac{1}{2}) \hat{\mathbf{x}} - az_{11} \hat{\mathbf{y}} + a (x_{11} + \frac{1}{2}) \hat{\mathbf{z}} & (24d) & \text{O IV} \\
\mathbf{B}_{181} &= -x_{11} \mathbf{a}_1 - y_{11} \mathbf{a}_2 - z_{11} \mathbf{a}_3 = -ax_{11} \hat{\mathbf{x}} - ay_{11} \hat{\mathbf{y}} - az_{11} \hat{\mathbf{z}} & (24d) & \text{O IV} \\
\mathbf{B}_{182} &= (x_{11} + \frac{1}{2}) \mathbf{a}_1 + y_{11} \mathbf{a}_2 - \\ & \quad (z_{11} - \frac{1}{2}) \mathbf{a}_3 = a (x_{11} + \frac{1}{2}) \hat{\mathbf{x}} + ay_{11} \hat{\mathbf{y}} - a (z_{11} - \frac{1}{2}) \hat{\mathbf{z}} & (24d) & \text{O IV} \\
\mathbf{B}_{183} &= x_{11} \mathbf{a}_1 - (y_{11} - \frac{1}{2}) \mathbf{a}_2 + \\ & \quad (z_{11} + \frac{1}{2}) \mathbf{a}_3 = ax_{11} \hat{\mathbf{x}} - a (y_{11} - \frac{1}{2}) \hat{\mathbf{y}} + a (z_{11} + \frac{1}{2}) \hat{\mathbf{z}} & (24d) & \text{O IV} \\
\mathbf{B}_{184} &= - (x_{11} - \frac{1}{2}) \mathbf{a}_1 + (y_{11} + \frac{1}{2}) \mathbf{a}_2 + \\ & \quad z_{11} \mathbf{a}_3 = -a (x_{11} - \frac{1}{2}) \hat{\mathbf{x}} + a (y_{11} + \frac{1}{2}) \hat{\mathbf{y}} + az_{11} \hat{\mathbf{z}} & (24d) & \text{O IV} \\
\mathbf{B}_{185} &= -z_{11} \mathbf{a}_1 - x_{11} \mathbf{a}_2 - y_{11} \mathbf{a}_3 = -az_{11} \hat{\mathbf{x}} - ax_{11} \hat{\mathbf{y}} - ay_{11} \hat{\mathbf{z}} & (24d) & \text{O IV} \\
\mathbf{B}_{186} &= - (z_{11} - \frac{1}{2}) \mathbf{a}_1 + (x_{11} + \frac{1}{2}) \mathbf{a}_2 + \\ & \quad y_{11} \mathbf{a}_3 = -a (z_{11} - \frac{1}{2}) \hat{\mathbf{x}} + a (x_{11} + \frac{1}{2}) \hat{\mathbf{y}} + ay_{11} \hat{\mathbf{z}} & (24d) & \text{O IV} \\
\mathbf{B}_{187} &= (z_{11} + \frac{1}{2}) \mathbf{a}_1 + x_{11} \mathbf{a}_2 - \\ & \quad (y_{11} - \frac{1}{2}) \mathbf{a}_3 = a (z_{11} + \frac{1}{2}) \hat{\mathbf{x}} + ax_{11} \hat{\mathbf{y}} - a (y_{11} - \frac{1}{2}) \hat{\mathbf{z}} & (24d) & \text{O IV} \\
\mathbf{B}_{188} &= z_{11} \mathbf{a}_1 - (x_{11} - \frac{1}{2}) \mathbf{a}_2 + \\ & \quad (y_{11} + \frac{1}{2}) \mathbf{a}_3 = az_{11} \hat{\mathbf{x}} - a (x_{11} - \frac{1}{2}) \hat{\mathbf{y}} + a (y_{11} + \frac{1}{2}) \hat{\mathbf{z}} & (24d) & \text{O IV} \\
\mathbf{B}_{189} &= -y_{11} \mathbf{a}_1 - z_{11} \mathbf{a}_2 - x_{11} \mathbf{a}_3 = -ay_{11} \hat{\mathbf{x}} - az_{11} \hat{\mathbf{y}} - ax_{11} \hat{\mathbf{z}} & (24d) & \text{O IV} \\
\mathbf{B}_{190} &= y_{11} \mathbf{a}_1 - (z_{11} - \frac{1}{2}) \mathbf{a}_2 + \\ & \quad (x_{11} + \frac{1}{2}) \mathbf{a}_3 = ay_{11} \hat{\mathbf{x}} - a (z_{11} - \frac{1}{2}) \hat{\mathbf{y}} + a (x_{11} + \frac{1}{2}) \hat{\mathbf{z}} & (24d) & \text{O IV} \\
\mathbf{B}_{191} &= - (y_{11} - \frac{1}{2}) \mathbf{a}_1 + (z_{11} + \frac{1}{2}) \mathbf{a}_2 + \\ & \quad x_{11} \mathbf{a}_3 = -a (y_{11} - \frac{1}{2}) \hat{\mathbf{x}} + a (z_{11} + \frac{1}{2}) \hat{\mathbf{y}} + ax_{11} \hat{\mathbf{z}} & (24d) & \text{O IV} \\
\mathbf{B}_{192} &= (y_{11} + \frac{1}{2}) \mathbf{a}_1 + z_{11} \mathbf{a}_2 - \\ & \quad (x_{11} - \frac{1}{2}) \mathbf{a}_3 = a (y_{11} + \frac{1}{2}) \hat{\mathbf{x}} + az_{11} \hat{\mathbf{y}} - a (x_{11} - \frac{1}{2}) \hat{\mathbf{z}} & (24d) & \text{O IV}
\end{aligned}$$

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